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USTS

STRESSES AND THRUSTS

STRESSES AND THRUSTS

A TEXT-BOOK FOR STUDENTS

(Being an Enlarged Edition of "Strains in Structures.")

BY
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PREFACE.

THE greater portion of this little volume is but a revised reprint of the treatise entitled "Strains in Structures," which was published some six years ago; but the inexactitude of that title, and the addition of several chapters which recently appeared in the *Building News*, rendering it still more incorrect, has led to it being altered to its present form. Intended mainly for the use of architectural students, it is hoped that it may be now found to be a satisfactory elementary work by all engaged in the study of "Stresses and Thrusts."

G. A. T. MIDDLETON.

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STRESSES AND THRUSTS.

CHAPTER I.

THE ELEMENTARY PRINCIPLES INVOLVED.

Before commencing the study of any subject demanding close logical reasoning, it is obviously necessary that the elementary principles involved should be thoroughly mastered, else the sound foundation is wanting upon which all else has to be erected

As in all branches of applied statics, the principles involved in the consideration of stresses and thrusts are those of—

(*a*) Moments round a point.

(*b*) The parallelogram, triangle and polygon of forces; and

(*c*) The centre of gravity.

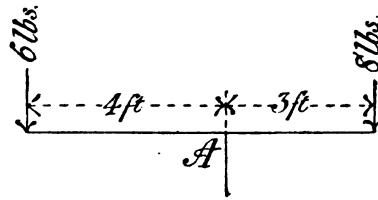
Let us take these in order.

The Principle of Moments is: that when a body is at rest, or in equilibrium, under the action of two or more forces, the tendency of all those forces which would go to cause rotation in one direction round a given point is exactly balanced by the tendency of all the forces which would go to cause rotation in the opposite direction.

This is best illustrated by a simple example. Let two weights of 8 lbs. and 6 lbs. respectively (see Fig. 1), be

supposed to rest at either end of a rod 7 ft. long, which is suspended from a point A 3 ft. from the larger and 4 ft. from the smaller weight. It will be seen by the diagram that the weight of 8 lb. is tending to cause rotation round A in the direction in which the hands of a clock move, while the tendency of the weight of 6 lb. is to cause rotation in the opposite direction round the same point. These tendencies, therefore, are opposite in direction, and they are also equal in amount, for it has been found, as the result of numberless experiments which have never

Fig. 1.



been known to fail (every time a butcher weighs a piece of meat he makes such an experiment), that the tendency to rotate is ascertained by multiplying the weight by the perpendicular distance from the point of rotation to the line in which the weight acts. In the case under consideration the weights act vertically downwards, so the perpendicular distances are the horizontal measurements along the rods, and the tendency of the larger weight to cause rotation is—

$$3 \text{ ft.} \times 8 \text{ lbs.} = 24 \text{ ft.-lbs.},$$

and similarly the tendency of the smaller weight to cause rotation is—

$$4 \text{ ft.} \times 6 \text{ lbs.} = 24 \text{ ft.-lbs.}$$

Thus 24 ft.-lbs. is in each case *the moment of the weight about*

the point of rotation. It will be observed that feet and pounds are here taken as the units in both cases; but inches or yards, and ounces or tons might just as readily have been adopted, so long as the same units were used throughout the particular calculation under discussion.

There is but one principle involved in the PARALLELOGRAM, TRIANGLE AND POLYGON OF FORCES. The first is shown in Fig. 2, in which two forces, A B and C B, are supposed to be acting in the directions shown by the arrows towards the point B, the line A B representing to

Fig. 2.

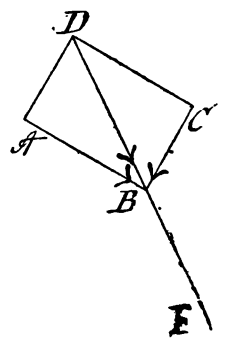
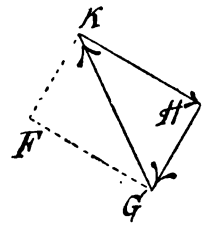


Fig. 3.



a scale of weights (so many pounds or tons to the inch) the magnitude, as well as the direction, of the force acting from A to B, and so on. Then, by completing the parallelogram A B C D, of which A B and C B are adjacent sides, it has been found that a single force acting in the direction of the diagonal from D to B, and equal to this diagonal in amount, as determined by scale measurement, would have the same effect upon a particle placed at B as would the two forces A B and C B acting together. This principle, again, like that of moments, has been proved

by numberless experiments which have never been known to fail.

Of course, if the force D B in amount, and acting from D to B in direction, be opposed by an exactly equal force E B, acting in exactly the opposite direction, from E to B, the particle at B will be held in equilibrium; and accordingly, if the three forces A B, C B and E B act upon the particle, the two forces A B and C B being known together to have the same effect as the one force D B, then also the particle is in equilibrium.

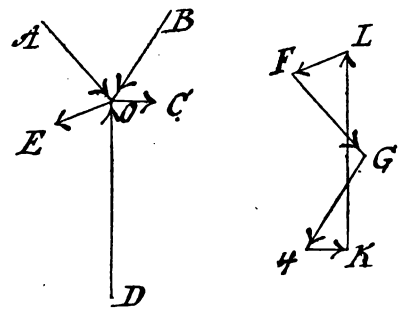
The triangle of forces is only used as a more simple way of expressing the same thing (see Fig. 3). In this the forces A B, C B and E B, which, as already said, hold the particle at B in equilibrium in Fig. 2, are represented by equal and parallel forces K H, H G and G K respectively, which, as will be seen by the arrows, act in sequence, as if a person were running from point to point, and form the closed triangle K H G. The remainder of the parallelogram of which they form half is dotted on the diagram, which parallelogram K H G F will be found to be similar in all respects to that in Fig. 2.

In the same way, the polygon of forces is but an extension of the triangle. Suppose the forces A O, B O, O C, D O and O E to be acting upon a particle at O (Fig. 4) in the directions designated by the arrows, then by drawing lines parallel and equal to these, in sequence, following the direction of the arrows, a closed polygon will be formed if the particle be held in equilibrium by these forces. In the case under consideration, the line F G represents the force A O, to which it is equal and parallel, G H represents B O, H K represents O C, and so on, the lines forming the complete closed polygon F G H K L F, representing in sequence each of the forces holding the particle at O in equilibrium.

THE CENTRE OF GRAVITY of a body is that point round which equate all the tendencies to rotate caused by the force of gravity acting upon the various particles of which the body is made up. Assuming a body of homogeneous substance and of uniform weight throughout (which is near enough to the truth in most cases for all practical purposes), this point has been determined for most simple-shaped bodies.

In cubes, rectangular or polygonal prisms having an even number of equal sides, spheres, cylinders, &c., the centre of gravity is at the geometric centre.

Fig. 4.



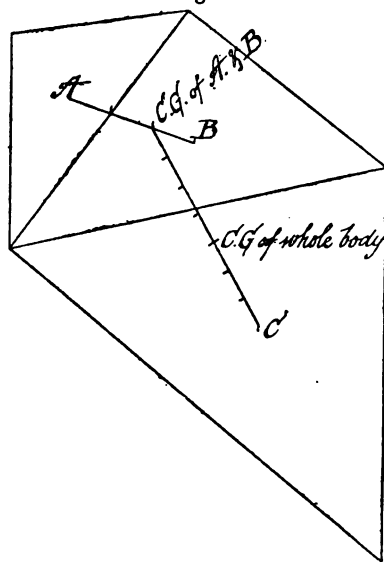
In triangles (by which is meant thin triangular prisms, as, of course, no body can have a centre of gravity which has not weight)—in triangles, then, the centre of gravity is found by bisecting each side and joining to the opposite apex, the point of intersection being the c.g., and this being two-thirds of the length of each of these lines from its apex.

In pyramids and cones the centre of gravity is situated at a point upon the right axis, distant three-fourths of its length from its apex.

Almost all practically occurring cases can be reduced to one or other of these forms, or to combinations of them.

When finding the centre of gravity of a combination of these forms, it is necessary to first find the c.g. of each simple form separately, and its weight (which may be supposed to be amassed at its c.g.), as is shown in the simple example given in Fig. 5. Here the weight of the

Fig. 5.



triangle A is supposed to be 1 lb., which is collected at its centre of gravity; and similarly the weight of the triangle B is supposed to be 2 lb., and that of the triangle C 4 lb., each collected at its c.g.

First, join A and B by a straight line, and as the sum of the weights A and B = 3 lb., therefore divide the line A B into three parts. The centre of gravity of the whole

figure A + B will be where shown, at a distance of one-third of the line A B from the end B, which is the more heavily weighted.

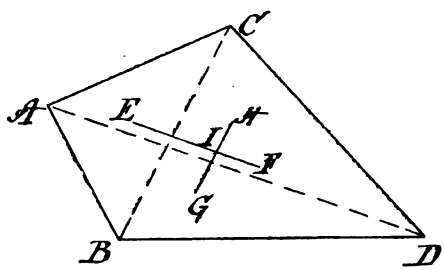
That this is true is proved by taking moments round this point, when—

$$1 \text{ lb.} \times \tfrac{2}{3} \text{ A B} = 2 \text{ lb.} \times \tfrac{1}{3} \text{ A B}$$
$$= \tfrac{2}{3} \text{ A B ft.-lb. in either case,}$$

showing that the tendencies to rotate round the c.g., due to gravity, equate.

Similarly, the whole weight of 3 lb. being now concentrated at this c.g., this point can be joined to the point C,

Fig. 6.



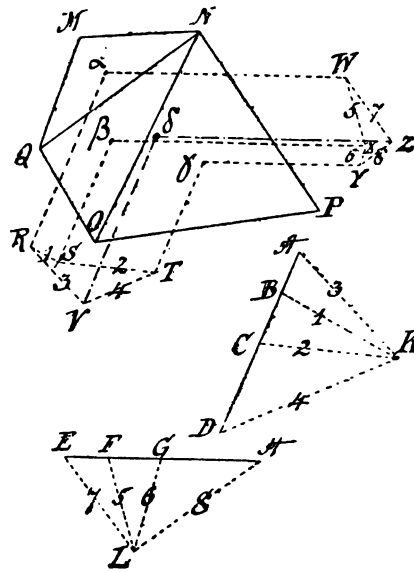
the joining line being divided into 7 parts (the sum of the weights being 7 lb.), and the c.g. of the whole body is then found to be 3 of these parts along this line as measured from C—the point which is most heavily loaded.

The centre of gravity of an irregular quadrilateral figure can be readily found diagrammatically, as shown in Fig. 6. The c.g. of the triangle A B C is found at E, and that of the triangle B C D at F, and the points E and F are joined. Similarly, the c.g. of the triangle A B D is found at G, and that of the triangle A C D at H, and the points

G and H are joined. The intersection of the lines E F and G H at the point I, gives the c.g. of the whole figure A B D C.

There is another diagrammatical way of determining the c.g. of a prism of irregular section, which is sometimes exceedingly useful, though apparently complicated. This

Fig. 7.



is exemplified in Fig. 7, in which the figure M N P O Q is an irregular polygon made up of the triangles M N Q, N Q O and Q O P, the weights of which are collected at α, β and γ respectively, which are the centres of gravity of the triangles.

Through α, β and γ draw three parallel lines—α R, β S

and γ T, and draw another line at side, by way of load line, also parallel, in which make A B equal to the weight located at α, to any scale; B C equal to the weight located at β, to the same scale; and C D equal to that located at γ. Then assume any pole K, and join K B, calling this line 1. Draw another line, also known as line 1, parallel to this one, between the lines α R and β S, from R to S. Similarly join K C, calling this line 2, and draw another line 2 parallel to it from S to T, cutting γ T in T; then join K A with line 3, and draw another line 3 parallel to it from R; and finally join K D with line 4, and draw another line 4 parallel to it from T, cutting line 3 in V. The c.g. of the whole figure will now lie somewhere in a line V S drawn through V parallel to α R, β S and γ T.

The operation has now to be repeated, drawing α W, β X and γ Y parallel to one another in some new direction (say, horizontally) and laying down a new load line E F G H parallel to them, in which E F represents to scale the weight located at α, F G the weight located at β, and G H that located at γ. Take a new pole L, and join L F in line 5 and L G in line 6, drawing other lines 5 and 6 parallel to them, connecting W X and X Y. Again join L E and L H in lines 7 and 8, and draw lines 7 and 8 respectively parallel to them from W and Y, meeting in Z. The c.g. of the whole figure must again lie in the direction of a line Z δ, drawn through Z parallel to α W, β X and γ Y, and therefore can only lie at the point δ, where V δ and W δ meet.

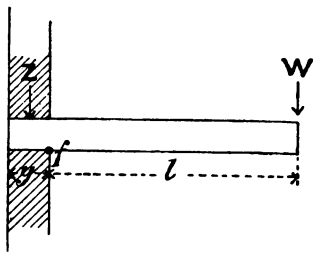
CHAPTER II.

LOADS ON SUPPORTS.

THE primary object of cantilevers, girders, bridges and roofs is to convey loads from their point or points of application to the points of support. In doing this various stresses are set up in the different members of these structures; but before the amount of these can be ascertained with any degree of accuracy, it is necessary, as will soon become apparent, to determine the proportion of the loads borne by each support.

In the case of cantilevers it is evident that, there being but one support, the whole weight must be borne by that support, whether the weight be fixed in one spot or divided into many smaller weights, or evenly distributed along the length of the cantilever.

Fig. 8.



The load itself, however, is, in the case of a cantilever, but a small proportion of what has to be carried by the support. Were not the weight W (see Fig. 8) counterbalanced by another weight Z, the cantilever would overturn about the fulcrum *f*. The weight Z necessary to prevent this can be readily ascertained, the cantilever now forming a lever of

the first order, by taking moments about *f*, remembering that Z may be considered to be applied at the centre of gravity of the surface pressed, or, in the example before us, at the centre of the wall. Thus, to produce equilibrium—

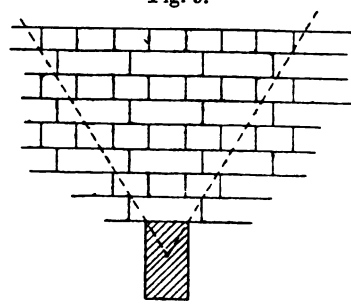
$$Z \times \frac{y}{2} = W \times l,$$

or

$$Z = \frac{2\,W\,l}{y}.$$

A considerably greater counterbalancing weight than Z, as ascertained above, should in all cases be provided for the sake of stability; but where a cantilever is built into a wall, this need not all lie directly over the cantilever, as all the bricks or stones which would be lifted in case of its overturning may be included in the load (see Fig. 9).

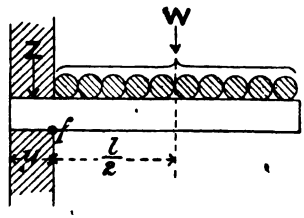
Fig. 9.



This allowance varies with the bond employed; but as a general rule all material which would be inclosed by an equilateral triangle having the centre of the cantilever for its apex (Fig. 9) may be safely considered as forming part of Z, no additional allowance being made for the adhesion of the mortar.

It is also a common plan, where a cantilever is expected to be subjected to a comparatively heavy end load, to carry it right through the wall and well into the building upon the other side, anchoring it down at its far end. The arm y of the lever being thus increased in length, the counterbalancing weight Z may be proportionally reduced.

Fig. 10.



When the load on the cantilever is distributed (Fig. 10), it may be considered to act at its centre of gravity, and the distance of its application from f becomes $\frac{l}{2}$. Then—

$$Z \times \frac{y}{2} = W \times \frac{l}{2},$$

or

$$Z = \frac{W l}{y}$$

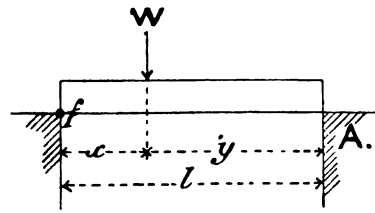
In either case the weight Z can be made up in many ways, as the exigencies of each case demand, and the total weight borne by the support becomes W* + Z.

When a fixed load is borne by a girder at the centre of its span, common sense tells us that half is carried by each support; but the proof of this depends again upon the principle of moments, and, as the proof is the same when W rests at any other spot than the centre, such a case is

* In this, as in most works on this subject, W is taken to represent a fixed load, or the total of a distributed load, w the load per unit of length (generally per foot), and l the length, in the same unit.

shown in Fig. 11. The girder may be considered as a lever, working round the fulcrum f to produce a certain

Fig. 11.

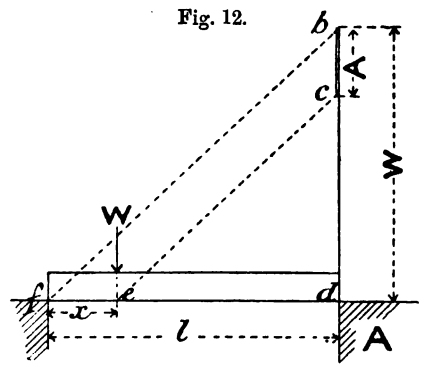


pressure on the right-hand abutment A. By taking moments round f, to produce equilibrium we must have—

$$\text{Pressure on A} \times l = \text{W} \times x;$$

$$\text{or Pressure on A} = \frac{\text{W}\,x}{l}.$$

Fig. 12.



This can be shown to scale graphically, as in Fig. 12. Set up W to a scale of so many tons or pounds to the inch over the abutment A. Join the extremity b of this line

to f. From e (a point vertically under the point of application of W) draw $e\,c$ parallel to $f\,b$; $c\,b$ will then represent the pressure on A to the same scale as that to which $d\,b$ represents W. The reason for this is that $d\,b\,f$ and $d\,c\,e$ are similar triangles, and so the sides $b\,d$ and $f\,d$ are cut proportionally by the parallels $f\,b$ and $e\,c$. That is—

$$b\,c : f\,e :: b\,d : f\,d,$$

or, in other words,

$$b\,c : x :: \mathrm{W} : l;$$

which is the same as saying,

$$b\,c \times l = \mathrm{W} \times x;$$

or

$$b\,c = \frac{\mathrm{W}\,x}{l};$$

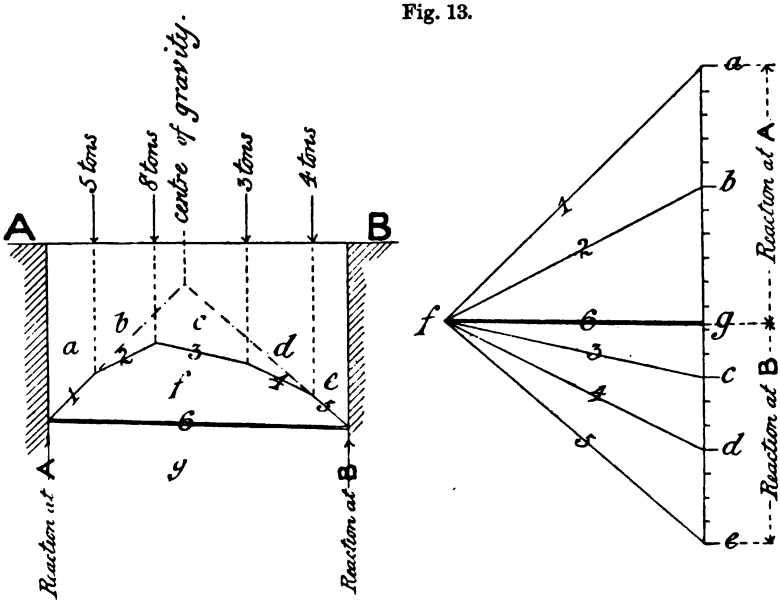
proving $b\,c$ to represent the pressure on A, as obtained by the previous reasoning.

Where there are several loads on a girder, each can be considered separately as above, and the totals on each abutment added together; or else all can be treated in one single diagram by a somewhat curious application of the polygon of forces.

Suppose a beam or girder, resting on the supports A and B (Fig. 13), to carry loads of 5, 8, 3 and 4 tons respectively where shown, the beam in this case being delineated by a single line. Place arrows pointing downwards where each load is applied, to denote the direction in which it acts, and arrows pointing upwards under the point at which the effect of the loads is transmitted to each abutment. Letter the spaces between these arrows in alphabetical order, commencing at the left-hand side, the space between the upward arrow under A and the downward arrow over the load of 5 tons being lettered a; that between the loads 5 and 8 tons being lettered b; and so on.

As the load of 5 tons lying between the spaces *a* and *b* acts *vertically downwards*, set off at the side, to a scale of weights as before described, a distance representing 5 tons on a line drawn *vertically downwards* from *a* to *b*.

Fig. 13.



Proceed in the same manner, setting off a distance representing 8 tons vertically downwards from *b* to *c*, to signify the load of 8 tons acting vertically downwards between the spaces *b* and *c*, and so on, taking care to make the letters on sketch and diagram tally for ease of reference. By this means all the loads on the girder can be set down to scale; and it is only then required to divide the whole vertical line *a e* into two parts to denote the proportions carried by the two abutments.

Assume any point *f* outside the line *a e*, and join *a f*, *b f*, *c f*, *d f* and *e f*. Drop vertical lines through the centres of each load (on the sketch), and also through the points A and B, where the loads are transmitted to the abutments. Assume any point in the line drawn thus through A, and from this point draw a line parallel to the line *f a* (on diagram) until it meets the vertical line through the load of 5 tons. Place another letter *f* on the opposite side of this new line to the space *a*, to represent the space in the centre of a polygon of which this line is one side, and it will be seen that there is a line between the spaces *f* and *a* upon the sketch parallel to the line *f a* on the diagram. From the termination of the line between the spaces *f* and *a* draw another between the spaces *f* and *b*, parallel to the line *f b* on the diagram, and terminating under the load of 8 tons, and proceed in this manner until the abutment B is reached. Join the extremities of these lines under the abutments A and B, inclosing *f* (by the line which is drawn double on sketch), and letter the space outside this and between the two upward arrows, *g*. The polygon now formed could be taken to represent in outline a truss such as would be in equilibrium under the assumed loads; and, by drawing through the point *f* on diagram a line *f g* parallel to the double line between the spaces *f* and *g* until it cuts the vertical line *a e*, the proportion of the total load borne by each abutment can be measured off, that carried by A being equal to the weight, as measured by the original scale of weights, represented by *a g*, and that carried by B being equal to the weight represented by *g e*.

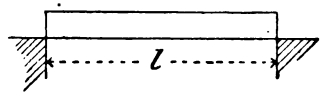
The loads and the proportions borne by the abutments are now represented by the closed polygon *a b c d e g* on the diagram, the pressures *e g* and *g a* on the abutments being supposed to act *upwards*, as indicated by the arrows

on the sketch in the first instance. This upward action or, more correctly speaking, passive resistance of the supports is an important factor in many calculations, and is generally known as *the reaction of the supports.* It is exactly equal at each support to the proportion of the load borne by that support, but is necessarily opposite to it in direction in order to cause equilibrium.

In Fig. 13, and in all other drawings throughout this work in which reciprocal diagrams are used, lines which have to be drawn parallel to one another upon sketch and diagram are similarly numbered.

This reaction of the supports is supposed to occur at the centre of that portion of the girder which rests on the support, and in all cases of calculation affecting beams

Fig. 14.



or girders it is usual to consider the span as from centre to centre of bearings, this being sufficiently near in most practical cases to be used instead of the correct centre of pressure of the reaction; and this distance is known as the "effective span." In diagrams this is usually shown as in Fig. 14, *l* being the "effective" span from centre to centre of bearings, and not the clear span, as would appear at first sight.

Half of a distributed load on a beam is evidently carried by each support, and when part of a beam bears a distributed load, while the remainder is unloaded, such distributed load must be treated as a number of small fixed loads, as shown in Fig. 13.

While upon this subject, it may be well to note that the

centre of gravity of a number of loads resting on a girder can be ascertained as shown by dotted lines in Fig. 13, by continuing the line between the spaces *f* and *a*, and that between the spaces *f* and *e*, until they meet, which they will do vertically over or under the centre of gravity of the loads.

CHAPTER III.

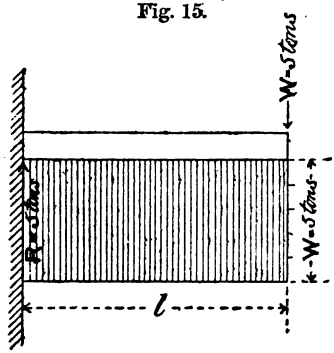
VERTICAL SHEARING STRESSES IN CANTILEVERS AND BEAMS.

THE opposition of the two forces spoken of in the last chapter—the load borne by a beam or cantilever, and the reactions of the supports—causes in the beam a stress similar to that caused in a piece of thread when exposed to the cutting edges of a pair of scissors, which stress is known as "shearing." Scissors, however, are made sufficiently powerful to overcome the resistance which thread offers to being shorn, while beams have to be designed to contain sufficient material in their webs (if they be of the flanged type) to resist the shearing effect of the load and reaction.

In the case of a cantilever carrying an end load (Fig. 15), the reaction R and the load W are equal to one another; but their lines of action, though parallel, are opposite in direction. From this it follows that the member which enables these two forces to oppose one another, and so to produce equilibrium, is subject at each point throughout its entire length to a shearing stress equal to either one of them—not to the sum of the two, for where two equal opposing forces of any kind meet, the stress produced is the measure of one of these forces only, because there could be none at all unless each force were opposed by an equal and opposite one, either active or passive. This can be shown graphically, as in Fig. 15, by setting down a line under W equal to W (here assumed as 5 tons)

to any scale of weights, and completing the shaded parallelogram. Ordinates drawn from beneath the cantilever to the lower limit of the parallelogram will give the shearing stress, to the same scale, at any point.

Fig. 15.



When a cantilever carries a distributed load, the shearing stress is only equal to the total load at the point of support, and uniformly diminishes as the reaction is taken up by one portion of the load after another, until at the end of the cantilever there is no shear at all. This can be shown graphically, as in Fig. 16.

When a cantilever extends beyond the point of application of the load, there is, of course, no shearing stress beyond that point, save that caused by the weight of the cantilever itself; and this, though occasionally of considerable importance in practical work, is rarely taken into account in purely theoretical investigations.

When a cantilever has to bear two or three different loads, the shearing diagrams have to be combined, as in Fig. 17, where a cantilever is represented as bearing a fixed load of 3 tons at one quarter of its length from the support, and a distributed load of 5 tons spread over the half of its length furthest from the support.

A fixed load on a *beam* causes a shearing stress at all points between its point of application and either support,

Fig. 16.

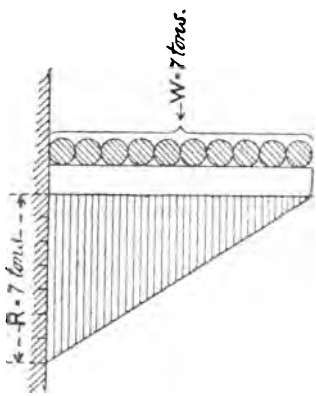
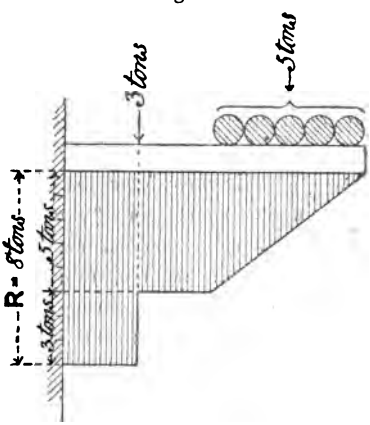


Fig. 17.

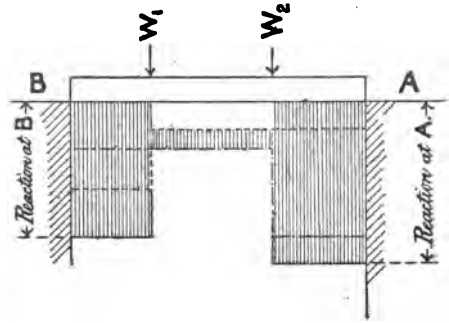


which is equal to the reaction at that support. The reasoning for this is the same as that in the case of the

cantilever: at every point between the point of reaction and the point of application of the load, the reaction is endeavouring to tear its way through the beam to reach the proportion of the load which equilibrates it.

In Fig. 18 the dotted outline denotes the shearing diagram due to W_1, the dot and stroke outline the diagram due

Fig. 18.

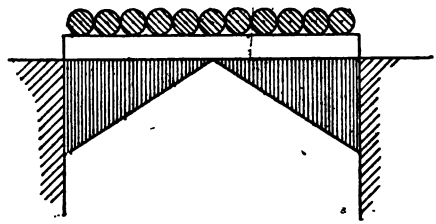


to W_2, and the shaded portion the combined diagram found by combining at all points the shears due to W_1 and W_2. Between the point of application of W_1 and the abutment B are found the proportions of both W_1 and W_2, which are borne by B, tearing their way towards that support, and being met by the total reaction at B, which is equal to their sum, causing a shear at all sections along that distance which is equal to the total reaction at B. Similarly, at all sections between the point of application of W_2 and the abutment A is found a shear which is equal to the total reaction at A. Between the points of application of W_1 and W_2, however, the proportion of W_1 which is borne by A is tearing towards A, and the proportion of W_2 which is borne by B is tearing towards B, these two neutralising one another, and leaving a

resultant shear equal to their difference at all sections between the points of application of W_1 and W_2.

Carrying this theory to its natural conclusion, it is found that, when two equal loads are placed at equal

Fig. 19.



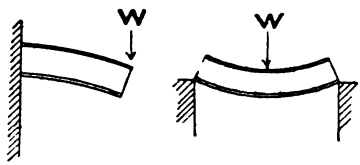
distances from the centre of a beam, there is no shear between their points of application, and that when a beam is subjected to an evenly-distributed load, there is none at the centre, the shears at the various sections being apportioned as is shown in Fig. 19.

CHAPTER IV.

STRESSES IN FLANGES.

In addition to the shearing stress which is calculated to be borne by the web, the flanges of flanged structures have to bear direct tension and compression. This is evident, as shown at Fig. 20, for a cantilever tends to

Fig. 20.

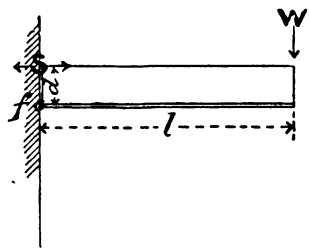


bend under a load so as to place the lower flange in compression and the upper in tension, while a beam tends to bend in such a way as to place the lower flange in tension and the upper in compression. It will be found upon investigation that in any vertical section the tension in the one flange is equal to the compression in the other, and that there is a line along the web at approximately half its depth (the actual position varying according to the design of the girder), known as the neutral axis, where there is no stress. In practice it is usual to consider that the whole of the tension and compression is borne by the flanges, and that the web has only to be made sufficiently strong to resist the shear.

In order to ascertain the stresses in the flanges, it is first necessary to assume the length (or span) l, and the

depth d. The exigencies of the particular case under discussion will generally determine l, while d is usually assumed as some fraction of l, varying, as circumstances will permit, from 1/12 to 1/20. These being given, the remainder is a matter of leverage. The double line in Fig. 21 shows the bent lever set in action by the weight W, acting at the end of a cantilever, and causing the tensional stress S in the top flange at the point nearest the support—where it is greatest. The fulcrum of this bent lever may be assumed at f, provided that the neutral axis be through the centre of the web, and round this point moments may be taken. The moment $S \times d$ must, to produce equilibrium, be equal to the moment $W \times l$, generally known as the *bending moment*. These can be equated thus—

Fig. 21.



$$S \times d = W \times l;$$

or, dividing both sides by d,

$$S = \frac{W l}{d}.$$

Of course, in the above it is necessary that whatever units be adopted for l and W, the same units must be adopted for d and S respectively. That is, if l be taken in inches, d must be taken in inches also; and if W be taken in tons, the stress S will also be ascertained in tons.

Correctly speaking, the long arm, l, of the lever should be taken along the neutral axis, the bending moment being equated by the moment of inertia, or the sum of resistances of the two flanges multiplied by their mean

distances respectively from the neutral axis—but the above explanation is the simpler one to follow, and produces practically the same result.

If it be required to determine the stress at any intermediate spot, it can be ascertained, as shown in Fig. 22, by assuming a bent lever round the fulcrum f', and taking the stress and bending moments round f'. Then—

$$S' \times d = W \times l';$$

or

$$S' = \frac{W \times l'}{d}.$$

As l' (Fig. 22) is less than l (Fig. 21), therefore S' is less than S. This proves that the greatest stress is at the support. Under the weight, l is reduced to *nil*, when

Fig. 22.

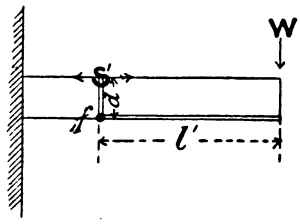
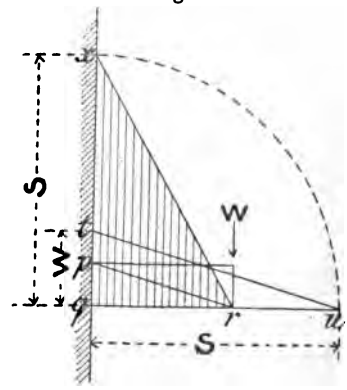


Fig. 23.



the stress also becomes *nil*; and the value of S will be found to vary uniformly from *nil* under the load to its greatest value at the support. The value of S will be the same whether it represent the tension in the top or the compression in the bottom flange, at whatever spot it may

be taken. It is only necessary to reverse the bent lever and to assume f in the top instead of the bottom flange to prove this.

The value of S in the formula $S = \frac{W\,l}{d}$ can be ascertained graphically, as in Fig. 23. Assuming $p\,q$ as the depth (d) and $q\,r$ as the length (l) of the cantilever, set up $t\,q$ equal to W, according to a scale of weights as previously explained. Join $p\,r$, and from t draw $t\,u$ parallel to $p\,r$, meeting $q\,r$ produced in u; then $q\,u$ is equal to S, for $p\,q\,r$ and $t\,q\,u$ being similar triangles, therefore—

$$q\,u : q\,r :: t\,q : p\,q;$$

or, multiplying the extremes and means inversely and equating,

$$q\,u \times p\,q = t\,q \times q\,r,$$

that is,

$$q\,u = \frac{t\,q \times q\,r}{p\,q} = \frac{W\,l}{d},$$

which proves that $q\,u = S$ to the same scale of weights to which $t\,q = W$.

By placing the compasses at q, and describing a quadrant with radius $q\,u$, $q\,x$ is made equal to S. Join $x\,r$, and then ordinates drawn from the bottom flange to the line $x\,r$ will denote the stresses in the flanges at any points to the same scale as that to which $t\,q$ represents W.

When a cantilever is subject to a distributed load, such load may be assumed to act at its centre of gravity (see Fig. 24). The bending moment is then but $W \times \frac{l}{2}$, the bent lever being as shown in Fig. 24, and the equation of equilibrium obtained by taking moments round the fulcrum f becomes—

$$S \times d = W \times \frac{l}{2};$$

or

$$S = \frac{W\,l}{2\,d}.$$

If it be required to ascertain the stress at any intermediate spot, it can be ascertained as shown in Fig. 25, that portion of the load only being considered which rests

Fig. 24.

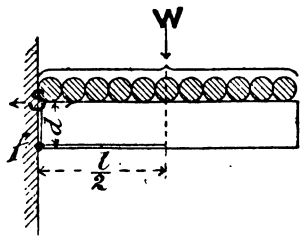
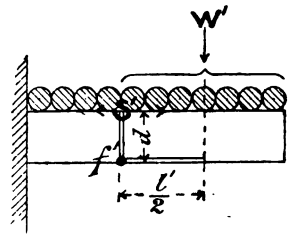


Fig. 25.



between the spot at which it is required to determine the stress and the end of the cantilever. The equation of moments would then be—

$$S' \times d = W' \times \frac{l}{2};$$

or

$$S' = \frac{W'\,l'}{2\,d}.$$

In this case, as in that of an end load, there is found to be no stress at the end of the cantilever, W and l having both diminished to *nil*; while the greatest value of S is obtained at the support p; but the intermediate variations are as the square of the distance from the extremity of the cantilever, instead of being uniformly as that distance.

The value of S in the formula $S = \frac{W\,l}{2\,d}$ can be ascer-

tained graphically, as shown in Fig. 26, in which the same letters have been adopted as in the similar Fig. 23, *r* now being placed under the centre of gravity of the total

Fig. 26.

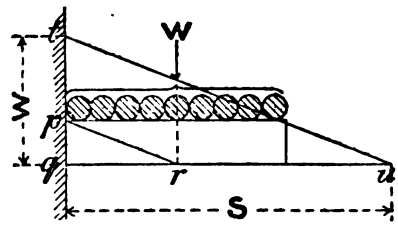
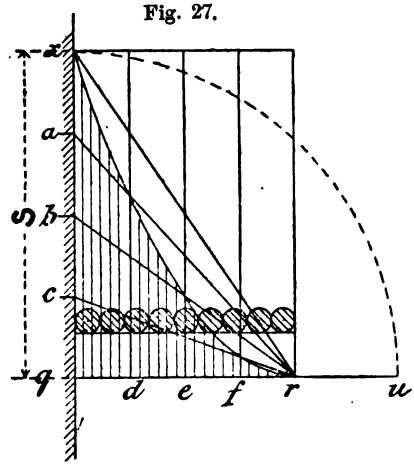


Fig. 27.



distributed load W. The same reasoning being followed as that used with Fig. 23, it is seen that in the similar triangles *p q r*, *t q u*,

$$qu : qr :: tq : pq;$$

or

$$qu \times pq = tq \times qr;$$

therefore,

$$q\,u = \frac{t\,q \times q\,r}{p\,q} = \frac{\mathrm{W} \times \frac{l}{2}}{d} = \frac{\mathrm{W}\,l}{2\,d};$$

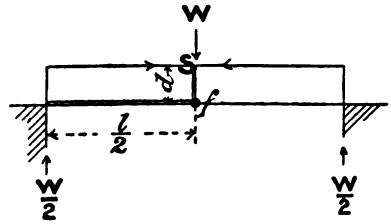
and therefore

$$q\,u = \mathrm{S}.$$

To determine the stresses at intermediate points, set up $q\,x$ equal to $q\,u$ (or S), as shown in Fig. 27. Between x and r set up a semi-parabola, having its apex at r. This may be accomplished by dividing $x\,q$ into any number of equal parts, as at a, b and c, and $q\,r$ into the same number of parts as at d, e and f. Join a, b and c to r, and through d, e and f erect vertical lines. Join x to r by a line passing through the points of intersection, thus tracing the outline of a semi-parabola. Ordinates drawn to this from the bottom flange will give the stresses at all intermediate points to the original scale of weights.

In flanged beams, the stresses are caused by the reactions of the supports, and not by the load itself, the greatest stress being under the load. When a beam has

Fig. 28.



to carry a fixed central load, the bent lever causing the maximum stress in the top flange is as shown in Fig. 28, f being the fulcrum, and the equation of moments—

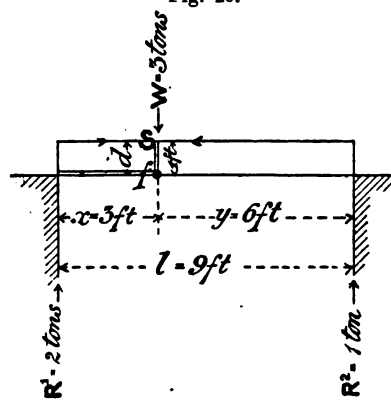
$$\mathrm{S} \times d = \frac{\mathrm{W}}{2} \times \frac{l}{2};$$

or

$$S = \frac{W l}{4 d}.$$

If the load be not centrally placed, the stress beneath it is found in precisely the same manner, the reaction at either support being taken to produce the bending moment: whichever be taken, the stress discovered to exist in the flange at the point under the load will be the same. An example is given in Fig. 29, where the

Fig. 29.



bending moment may be taken to be either $R^1 \times x$ or $R^2 \times y$; in either case, 6 foot-tons. In other words—

$$S \times d = R^1 \times x, \text{ or } R^2 \times y;$$

$$\therefore S = \frac{R^1 \times x}{d}, \text{ or } \frac{R^2 \times y}{d}$$

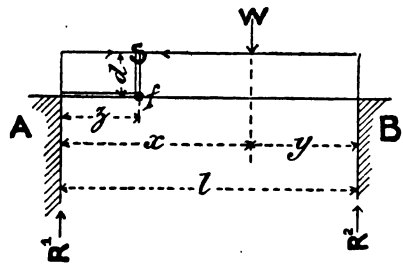
$$= \frac{2 \times 3}{1}, \text{ or } \frac{1 \times 6}{1}$$

$$= 6, \quad \text{or} \quad 6.$$

The stresses at intermediate points are found as shown

in Fig. 30, R^1, the reaction at the abutment A causing the stresses at all points between that abutment and the point of application of W, and R^2 causing the stresses at

Fig. 30.



all points between the abutment B and the point of application of W. In Fig. 30 it is assumed that the flange stresses are required to be ascertained at a point at which

$$S \times d = R^1 \times z;$$

or

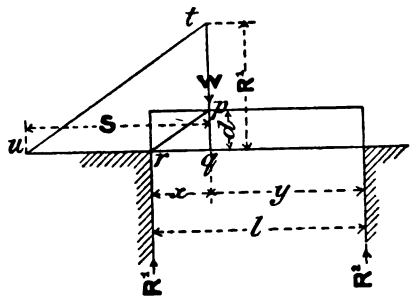
$$S = \frac{R^1 \times z}{d}.$$

This value of S is evidently greatest when z is greatest—that is, when S is required under the load—and least when z is least, viz. at the abutment, when z is *nil*, and S consequently is *nil* also. The stress varies uniformly according to the value of z.

The value of the stresses under a fixed load, whether it be central or otherwise, can be ascertained graphically, as shown in Fig. 31, in which the same letters have been used as in Figs. 23 and 26. From the bottom flange under the load, tq is set up equal to R^1 to any scale of weights, p and r are joined by a straight line, and through

t a line $t\,u$ is drawn parallel to $p\,r$. S is now denoted by $q\,u$ to the same scale as that to which $q\,t$ was set up.

Fig. 31.



This depends upon similar reasoning to that adopted before, for $p\,q\,r$ and $t\,q\,u$ being similar triangles,

$$q\,u : q\,r :: t\,q : p\,q;$$

or

$$q\,u \times p\,q = t\,q \times q\,r;$$

$$\therefore\ q\,u = \frac{t\,q \times q\,r}{p\,q}$$

$$= \frac{\mathrm{R}^1 \times x}{d};$$

and therefore $q\,u = \mathrm{S}$, for this agrees with the result arrived at above when considering Fig. 29.

To determine the stresses at intermediate points, set up $q\,m$ vertically from the bottom flange under the load equal to $q\,u$ or S. (See Fig. 32.)

Join $m\,r$ and $m\,n$, and then ordinates set up from the bottom flange to $m\,r$ and $m\,n$ will give the stresses at any intermediate points.

It is easiest to ascertain the stresses in a flanged beam due to a distributed load by making an assumption. Two

or three different assumptions will lead to the desired result; but perhaps the easiest to comprehend is that shown in Fig. 33. The proportion of the load on the

Fig. 32.

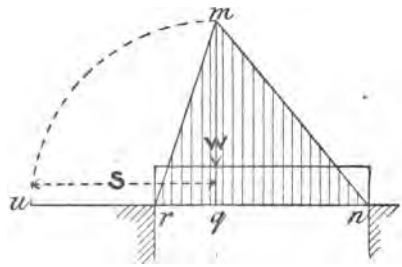
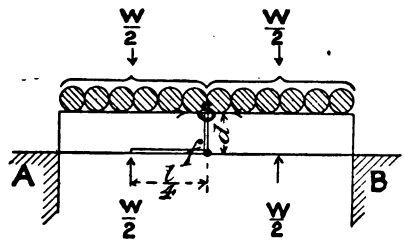


Fig. 33.



half-beam nearest to the abutment A, instead of reacting at A is assumed to react at its centre of gravity.

By this means the lever causing stress at the centre becomes as shown, and—

$$S \times d = \frac{W}{2} \times \frac{l}{4},$$

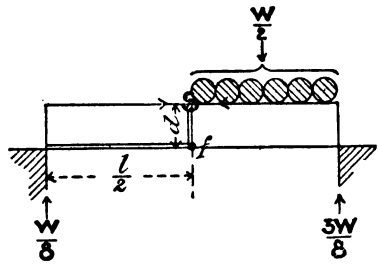
or

$$S = \frac{W\,l}{8\,d}.$$

The same result can be arrived at by supposing each half

of the load $\left(\frac{W}{2}\right)$ to act at its centre of gravity as a fixed load. Then the reaction at the further abutment due to this half-load is $\frac{W}{8}$ (see Fig. 34), causing stress at the

Fig. 34.



centre with a leverage of $\frac{l}{2}$, this stress being half the stress which would be produced were the girder loaded throughout its whole length, or $\frac{S}{2}$, as a similar stress would be produced by the load lying on the other half of the girder. Thus,

$$\frac{S}{2} \times d = \frac{W}{8} \times \frac{l}{2},$$

or

$$\frac{S}{2} = \frac{W\,l}{16\,d},$$

or

$$S = \frac{W\,l}{8\,d},$$

the same result as before ascertained.

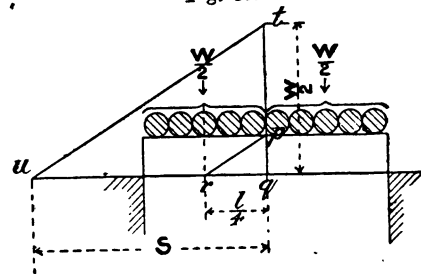
The stresses at intermediate points can be similarly found by assuming the portion of the load on either side of the point at which the stress is required to be

found to be a fixed load acting at its centre of gravity, ascertaining the stresses produced by each of these loads at the required point, and summing them.

The stress is greatest at the centre, and is *nil* at the supports, but varies between these points in a compound ratio, which is sufficiently indicated in the last paragraph.

These stresses can be ascertained graphically, as shown in Fig. 35, in which the same letters have been used as in

Fig. 35.



the previous examples (Figs. 23, 26, and 31), $t\,q$ being set up over the centre equal to $\frac{W}{2}$ to scale of weights, and $q\,r$ being taken equal to $\frac{l}{4}$. Then $q\,u$ is equal to the stress in the centre. The reasoning is exactly similar to that adopted before, for $p\,q\,r$ and $t\,q\,u$ being similar triangles,

$$q\,u : q\,r :: t\,q : p\,q,$$

or

$$q\,u \times p\,q = t\,q \times q\,r;$$

$$\therefore q\,u = \frac{t\,q \times q\,r}{p\,q} = \frac{\frac{W}{2} \times \frac{l}{4}}{d} = \frac{W\,l}{8\,d} = S.$$

The intermediate stresses are more readily ascertained geometrically than arithmetically (see Fig. 36). Set up

$q\ m$ vertically over the centre of the bottom flange equal to $q\ u$ or S as previously ascertained, and describe the parabola as shown (in the same manner as explained in

Fig. 36.

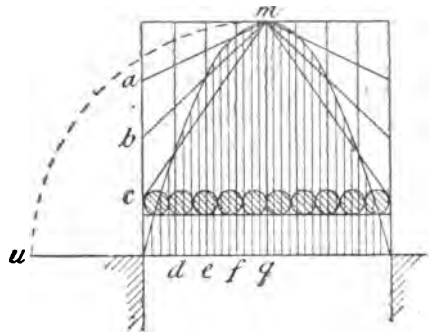
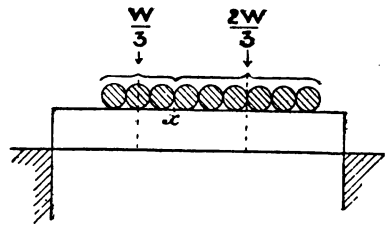


Fig. 27). Ordinates drawn from the bottom flange to the outline of this parabola will give the stresses at any intermediate points.

When a load is distributed over only a part of a girder, the remainder being unloaded, it is usual to assume the load to be a fixed load acting at its centre of gravity. This can be done without great error when the load covers only a small portion of the length of the girder, while such error as occurs is on the side of safety; but when the load and the proportion of the girder which it covers are considerable, it is wisest to ascertain the stresses at all points correctly, according to the method explained later on with Fig. 43, or else as shown in Fig. 37, by taking several points as x, assuming the load lying on each side of x to act at its centre of gravity, ascertaining the stresses at x due to each of these parts of the load, and summing them. By doing this at several points, a stress outline can be set up which will delineate the stresses at all points, as is

done in Figs. 32 and 36. It will be noticed, however, that the curved outline will only extend over that portion of the beam which is loaded; over the unloaded portion the outline will be exactly the same as it would have been

Fig. 37.



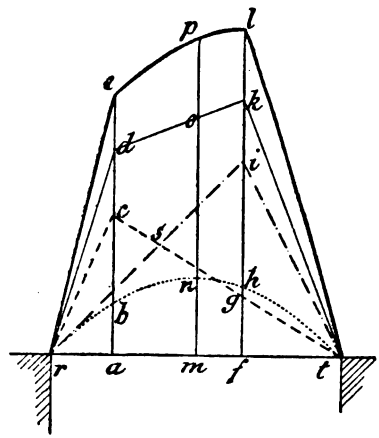
had the load been a fixed load acting at its centre of gravity. The error of assuming the load to be fixed is thus seen to be slight, and only to affect the portion of the beam which is loaded, while its rectification is only of importance as saving the employment of unnecessary material in the flanges directly under the load.

When, as often happens, a beam has to carry several loads at different points, the stresses due to each have to be ascertained separately, and then combined at all points before a girder can be designed to resist them. To do this arithmetically, by ascertaining the stresses at several given points from each load in succession, and adding these stresses together to ascertain the total stress at each of these points, is tedious in the extreme. Graphically considered, the problem is an easy one.

In Fig. 38 an outline of short dashes (*r c t*) represents the stress outline due to one fixed load, the dot-and-stroke outline *r i t* represents the stress outline due to another fixed load, and the dotted outline *r b n h t* represents the stress outline due to a distributed load, such as the weight of the girder itself. Of the triangles *r c t* and *r i t*, the

portion *r s t* is common to both—which means that the stresses which it represents occur twice over, and that the figure *r c s i t* plus the figure *r s t* represents the stresses

Fig. 38.



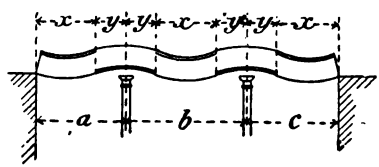
due to the two fixed loads. The triangle *r s t* has therefore to be combined with the figure *r c s i t* to represent these stresses in one figure. This can be done by making *i k* equal to *f g*, and *c d* equal to that portion of the line *a c* which lies within the triangle *r s t*, and by joining *r d k t*. The figure *r d k t i s c r* can now be readily proved, by the theory of triangles on equal bases and with equal altitudes, to be equal to the triangle *r s t*. In other words, the figure *r d k t* represents the total stresses due to the two loads.

The stress outline *r b n h t*, due to the distributed load, can similarly be added to this combined diagram by making *d e* equal to *a b*, *o p* equal to *m n*, and *k l* equal to *f h*, and by joining *r e p l t* in the firm curved outline shown, which outline represents the total stresses at all points due to the two fixed loads and the distributed load.

In all cases yet investigated it has been assumed that the girders merely rest upon the supports without being fixed in any way. When the ends of girders are fixed, they are placed under an entirely new set of conditions; but it is so difficult to absolutely fix them over single spans that they are almost invariably assumed to be supported only even when held in position by bolts or by a superincumbent weight. When, however, girders are so built as to form one continuous structure over several spans, the portion over each central span becomes absolutely fixed at its ends.

In Fig. 39 an example is given of a girder which is continuous over three spans, *a*, *b* and *c*. Of these spans

Fig. 39.



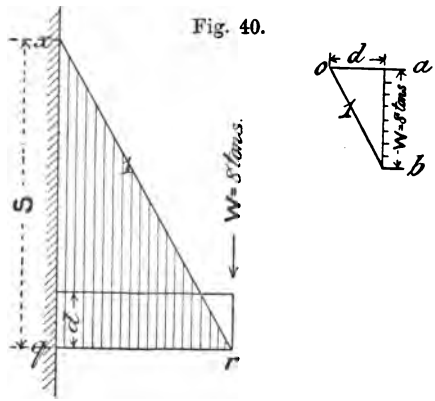
the portions *x x x* are girders with the top flange in compression and the bottom in tension; while the portions *y y y y* are cantilevers with the top flange in tension and the bottom in compression. The stresses on *x* are found as usual for girders, with a span equal to *x*. One of the abutments of *x* is the end of *y*, which carries its proportion of the load on *x* as an end load, besides any load to which it may itself be subject, the stresses being ascertained in the usual manner for cantilevers. The greatest stress will, under almost all conditions, be found to be over the intermediate supports, while there will be no stresses at the points where the ends of *x* rest on the ends of *y* (which points are called "points of contraflexure").

It has been by the careful adoption of this principle of

continuous girders that such immense bridges as that over the Forth at Inchgarvie have been rendered possible.

The stresses in the flanges of supported (i. e. not fixed) girders can be ascertained at all points in an exceedingly simple manner by means of a reciprocal polar diagram, similar in principle to that shown in Fig. 13. Taking the simplest case—that of a cantilever with an end load—and assuming this load W to be equal to 8 tons, acting vertically downwards, set this 8 tons to a scale of weights vertically downwards from a point *a*. (See Fig. 40.) From *a*

Fig. 40.



set off *a o* horizontally, equal to *d*, the depth of the cantilever, and join *o b*. From *r*, a point in the bottom flange of the cantilever immediately under the load, draw *r x* parallel to *o b* on the diagram. Ordinates from *q r* to *x r* will give the value of the stresses in the flanges at any point.

The proof of this is simple, for the triangles *x q r*, *b a o* are similar to one another, and therefore—

$$xq : qr :: ba : ao,$$

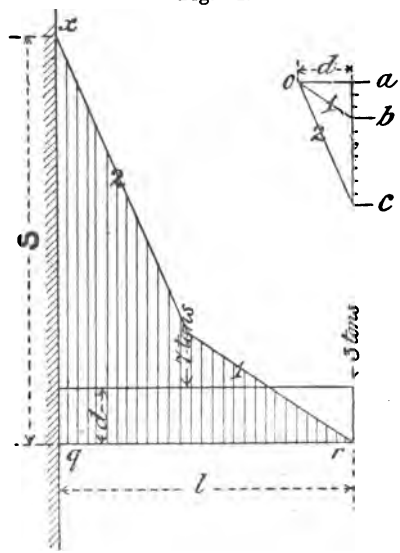
or $$xq \times ao = ba \times qr,$$

or $$xq = \frac{ba \times qr}{ao} = \frac{Wl}{d},$$

proving that $x\,q = S$ according to the recognised formula. Moreover, the triangle $x\,q\,r$ thus obtained is identical with the triangle $x\,q\,r$ as ascertained in Fig. 23.

In a simple case like the foregoing the advantages of this method are not very conspicuous; but more complicated cases are much simplified by its adoption. Suppose, for example, that the cantilever carries two fixed loads of 3 and 7 tons respectively at different points (see Fig. 41),

Fig. 41.



acting vertically downwards. According to a scale of weights, set vertically down from a point a the distance $a\,b$, equal to 3 tons, and from b set down $b\,c$, equal to 7 tons. From a set off $a\,o$ horizontally, equal to d, the depth of the cantilever. Join $o\,b$ and $o\,c$.

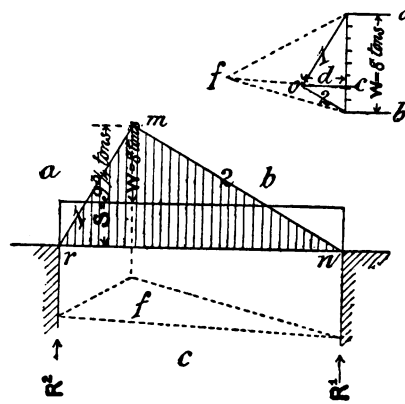
From the point r vertically under the load of three tons, and in the bottom flange of the cantilever, draw a line

parallel to *o b* until a vertical line through the centre of gravity of the load of seven tons is reached; and from that point draw a line parallel to *o c*, meeting the abutment at *x*. Ordinates drawn from *q r* to the outline from *x* to *r* will give, to the scale of weights employed in the diagram, the value of the stresses in the flanges at all points; and a little consideration will show that the figure *x q r* merely contains the sum of the triangles which would have been obtained had each of the weights been considered separately. Cases in which three or four, or any greater number of loads have to be considered, can be treated in a precisely similar manner, until, by considering a distributed load to be an aggregate of an infinite number of infinitesimal loads, an outline similar to that shown in Fig. 27 can be arrived at. In order to obtain an accurate result, however, in this, as in all other systems in which diagrams are employed, the most absolutely perfect draughtsmanship is essential. In important calculations it is therefore desirable to employ a large scale and a fine line, so as to reduce the risk of errors to a minimum.

A simple application of this system to a supported girder can now be taken, in which a weight of eight tons acts vertically downwards at some fixed point. (See Fig. 42.) From any point *a* set *a b* vertically downwards, equal to eight tons to any scale of weights. From *b* set *b c* vertically upwards, equal, to the same scale, to R^1, the reaction of the right-hand abutment. Then *c a* will be equal to R^2, the reaction of the other abutment: these reactions being ascertained by the dotted polar and funicular diagrams, as explained in connection with Fig. 13. From *c* set *c o* horizontally equal to *d*, the depth of the girder. Join *o a* and *o b*. From *r*, the point in which the bottom flange of the girder meets the left-hand abutment, draw *r m* parallel to *o a*, and from *n* draw *n m* parallel to

o b. If accurately drawn, *r m* and *n m* will meet in *m*, vertically over the centre of gravity of the weight W; and ordinates drawn from *r n* to the outline *r m n* will

Fig. 42.

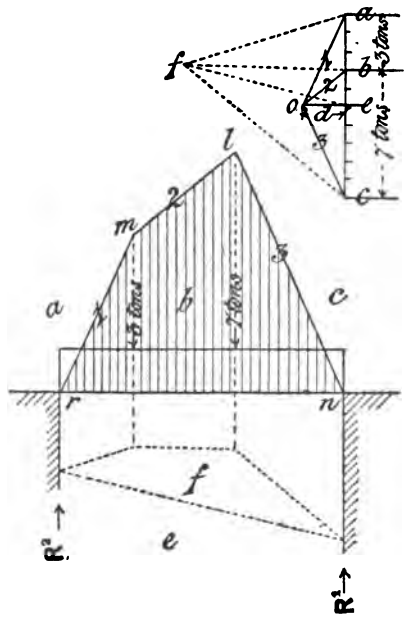


give the value of the stresses in either flange at any point, to the same scale of weights as that employed in setting down the diagram. Very little consideration will show that the triangle *r m n* thus obtained is identical with the triangle *r m n* obtained in Fig. 32. The proof is obvious and need not be repeated.

The advantages of this method of computing stresses are, in the girder as in the cantilever, more considerable in complicated than in simple cases. Suppose that the girder carry two loads of 3 and 7 tons respectively at different points. (See Fig. 43.) From a point *a* set vertically downwards *a b* to any scale of weights equal to 3 tons, from *b* set down *b c* equal to 7 tons, and from *c* set upwards *c e* equal to R^1, the reaction at the right-hand abutment; *e a* will then be equal to the reaction at the

other abutment. These reactions, as before, are ascertained by the dotted polar and funicular diagrams. From *e* set off horizontally *e o* equal to *d* the depth of the girder,

Fig. 43.



and join *o a*, *o b* and *o c*. From *r* draw *r m* parallel to *o a* until a vertical line through the centre of gravity of the load of 3 tons is reached at *m*. From *m* draw *m l* parallel to *o* till a vertical line through the centre of gravity of the load of seven tons is reached at *l*. From *l* draw *l n* parallel to *o c*. If the drawing be accurate, the point *n* should correspond with the corner of the right-hand abutment. Ordinates from *r n* to the outline *r m l n* will give

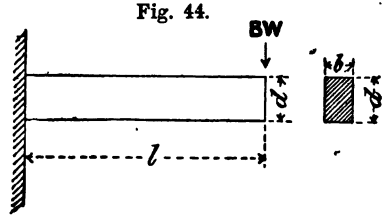
the values of the stresses in either flange at any point to the scale of weights employed. It will be found on trial that the polygon *r m l n* corresponds with that obtained by summing the triangles arrived at by considering each of the loads separately, as explained in connection with Fig. 38. The system is applicable to any number of loads, until, by considering a distributed load as an aggregate of many small loads, an outline similar to that shown in Fig. 36 would be arrived at.

CHAPTER V.

RECTANGULAR BEAMS.

THE loads which beams of simple rectangular section are capable of sustaining without failure (generally known as their "B.W.," or "breaking weights"), cannot be ascertained with such mathematical accuracy as can the stresses in flanged structures; this being mainly due to the varying strength of different specimens of the same material. Beams of certain small sections and of slight span have been experimented upon, an average struck of the results, and a system of simple proportion adopted to arrive at the strength of other beams of the same material, the results having on experiment proved to be approximately correct. The constant originally adopted was a load under which a beam of 1 in. square section and 1 ft. long would break were it applied at the centre, and the constants here given have been deduced from this arithmetically.

Fig. 44.



In the case of a rectangular cantilever bearing an end load (see Fig. 44), in which cantilever, b equals the breadth, d the depth and l the length, it is fair to assume that—

B W is proportional to the sectional area $= b\,d$,

also that B W is proportional to the leverage $= \frac{d}{l}$,

and that B W is proportional to the material $=$ K,

a constant determined by experiment. Therefore B W may be taken to equal the product of these three things to which it is proportional, or—

$$\text{B W} = b\,d \times \frac{d}{l} \times \text{K}$$

$$= \text{K}\frac{b\,d^2}{l}.$$

This equation holds only when b, d and l are all taken in inches, B W and K being both taken according to the same denomination. (As a rule, K is taken in cwts., with the result that B W is also found in cwts.) Some authors take L in feet instead of l in inches, and slightly vary the formula, using the value for K originally arrived at as explained above, and not the value here given. Students must not become confused between different authorities in this matter, for the result achieved is practically the same in all cases. The reasons why the formulæ given in this work have been chosen are their simplicity, the ease with which they can be remembered, the fact of the same denomination (inches) being used throughout, and of their running on "all fours" with the formulæ given in Chapter IV. for ascertaining the stresses in flanges. The values of K to suit these equations are as follows:

K	for Baltic fir	=	13 cwt.
K	„ English oak	=	15 „
K	„ Cast iron	=	46 „
K	„ Wrought iron	=	68 „

It is not necessary in ordinary practice to be acquainted with the values of K for any other material.

The equation given above for a cantilever with an end load varies under different conditions according as the leverage varies, and thus:

B W when distributed along the length of a cantilever .. .. $= 2\,K\frac{b\,d^2}{l}$.

B W when situated centrally on a beam $= 4\,K\frac{b\,d^2}{l}$.

And B W when distributed over the length of a beam .. .. $= 8\,K\frac{b\,d^2}{l}$.

It must be carefully remembered that the *breaking* weight, or that under which a structure composed of average material would succumb, is what is ascertained by the above, the "safe load," or that which it would be safe to allow such a structure to carry, being some proportion of this. It is usual to consider that in timber structures the safe load is $\frac{B\,W}{6}$, and in metal structures $\frac{B\,W}{4}$.

It will be noticed, on careful investigation, that in all these cases the *bending moment* equates with $K\,b\,d^2$; and this is true for all rectangular beams, however loaded. Thus, with one or even several loads, wherever placed, it is only necessary to equate the greatest bending moment with $K\,b\,d^2$ in order to ascertain its necessary section. Of course, that this may be a safe section, the actual load must be multiplied by the factor of safety to raise it to the B W before the bending moment is ascertained. It is usual in such cases either to assume the value of d, or else to take $b = \frac{5\,d}{7}$.

Efforts have been made to adapt the above formulæ to other sections than the plain rectangle. In flanged structures, at least, this is unnecessary, for, given the section, the safe stress on either flange, according to the material, is known, and from the stress formula, W, then the only unknown quantity and representing the "safe" load, can be ascertained. Under these circumstances it is not wise to burden the mind with unnecessary formulæ.

Theoretically, the correct forms for beams of rectangular section should be those given in Figs. 45 to 52. Practically, it is generally found to be more economical to make them rectangular in all directions, and of their maximum section uniformly throughout.

Correct theoretical forms for rectangular beams to assume.

Cantilever with end load.

Fig. 45.

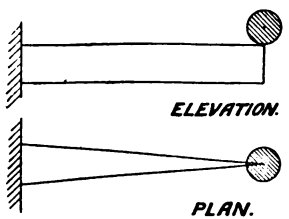
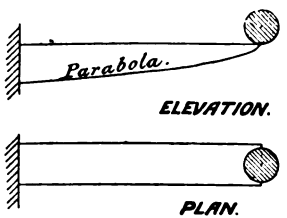


Fig. 46.



Cantilever with distributed load

Fig. 47.

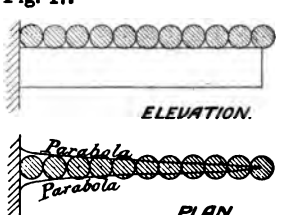
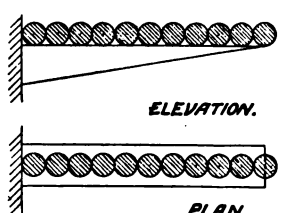


Fig. 48.



Beam with single load.

Fig. 49.

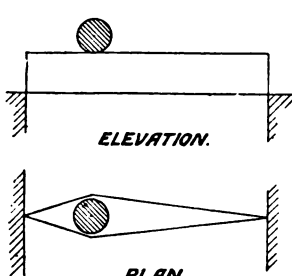
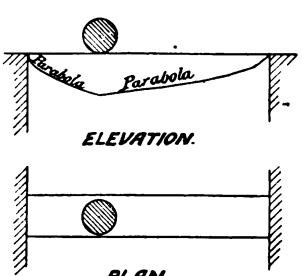


Fig. 50.



Beam with distributed load.

Fig. 51.

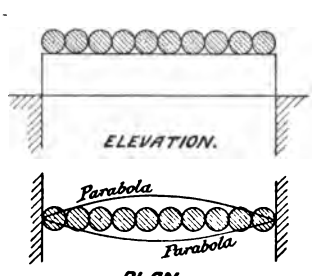
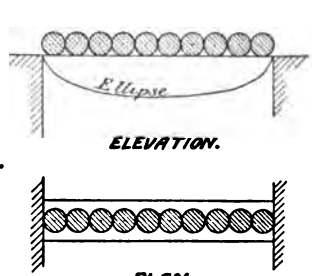


Fig. 52.

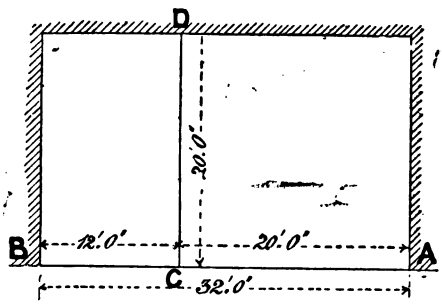


CHAPTER VI.

METHOD OF DESIGNING A WROUGHT-IRON GIRDER.

BEFORE proceeding to the consideration of framed structures, it would be well to work out in detail an example of the method to be followed in designing a girder under given conditions. The assumed plan (Fig. 53) will serve this purpose. A girder of 32 ft.

Fig. 53.



span from A to B carries a wall weighing 1¼ tons per foot run, and also helps to support the short girder C D of 20 ft. span, which carries a wall weighing 1 ton per foot run. These loads are assumed to include the weights of the girders themselves.

In a case like this it is necessary to consider the case of the smaller girder first, and to commence by assuming its depth. This is generally taken at anything between

$\frac{1}{12}$ and $\frac{1}{20}$ of the span, so that a girder 15 in. deep will suffice. The stress at the centre in either flange may be found by the formulæ

$$S = \frac{W\,l}{8\,d} = \frac{20 \text{ tons} \times 20 \text{ ft.}}{8 \times 1\frac{1}{4} \text{ ft.}} = \frac{400 \text{ ft.-tons}}{10 \text{ ft.}} = 40 \text{ tons.}$$

Set this up to scale over the centre of a line representing the girder (Fig. 54), and complete the parabolic outline of the stresses at intermediate points, as explained in Chapter IV.

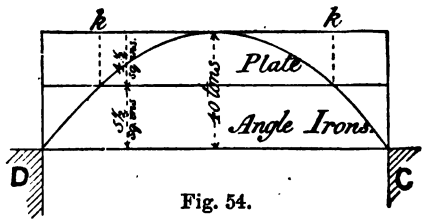


Fig. 54.

Ordinary wrought iron being capable of safely sustaining a compressional stress of four tons per square inch, it is evident that 10 sq. in. are required in the sectional area of the top flange at the centre to resist a stress of 40 tons. The rivets may be included in this area, as they help to transmit the compression through their shanks.

The angle irons are usually included in the flanges for calculations of area to resist stress. Small L-irons would here be used, say 3 in. by 3 in. by $\frac{1}{2}$ in., being $2\frac{3}{4}$ sq. in. each in sectional area. Thus the two angle irons attached to the top flange contain together $5\frac{1}{2}$ sq. in. of metal. The remaining $4\frac{1}{2}$ in. required in the flange can be provided by a 9 in. by $\frac{1}{2}$-in. plate. The proportion of width to depth thus obtained is fairly good; but theoretically there are no laws to fix this, it being mainly a matter of convenience.

The depth of 40 tons (Fig. 54) should be divided proportionally to the area contained by the L-irons and plate

respectively, and right lines drawn through. A plate diagram is thus drawn over the stress diagram, and should at all points include the latter. This could be accomplished by stopping the plate at *k k*; but it is hardly worth while to do so on so small a girder, and there would be practical disadvantages in reducing the width of the flanges near the abutments to that covered by the L-irons only.

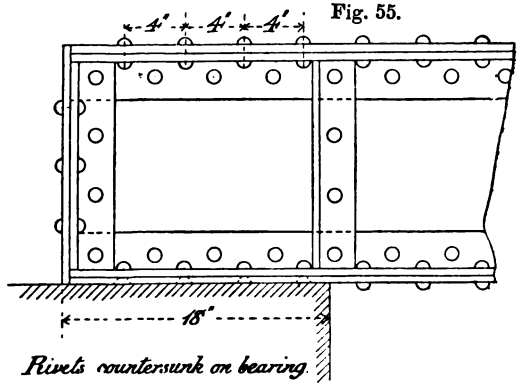
Having thus determined the section of the top flange, it will be found that the bottom flange may be made similar to it (the material used being wrought iron). The rivet *holes* here have to be deducted, as the tensional stress has to be conveyed past the rivets by the remaining metal. Two rivet holes occur in each sectional area of flange, the rivets being necessary in order to attach the angle irons to the plates; and another rivet is used to attach the angle irons together through the web. Though this is not in the same transverse section, it is considered in practice safer to deduct the area of the hole. If $\frac{7}{8}$-in. rivets be employed, the total area thus deducted from the bottom flange is $1\frac{7}{8}$ in., leaving an effective area of 10 in. $- 1\frac{7}{8}$ in. $= 8\frac{1}{8}$ in. to resist a tensile stress of 40 tons. This it is more than capable of doing, as wrought iron will bear 5 tons per square inch safely in tension. Thus, if the top flange be designed to meet the compression brought to bear upon it, a similarly constructed flange will suffice for the bottom.

There are many practical advantages in building both flanges alike, the principal one being that there is no risk of error through the workmen, in setting the girder, placing the wrong flange uppermost.

The shear in the web is but little, being only 10 tons over either abutment, and diminishing towards the centre. As the iron will stand 4 tons per square inch in shear safely, only $2\frac{1}{2}$ sq. in. of metal is required in the web,

and this is much more than supplied by its being made $\frac{3}{8}$ in. thick, thinner than which would be valueless against possible corrosion and buckling. Stiffeners would also be

Fig. 55.



provided as precautions against buckling, and would consist of L-irons of the same size as those used to attach the flange plates to the web—3 in. by 3 in. by $\frac{1}{2}$ in. In small girders, such as the one we are considering, packing pieces of scrap iron have to be inserted under the stiffeners to save a troublesome fold round the angle irons of the flanges. The stiffeners are placed about 4 ft. apart, their actual position being determined by the exigencies of the case, one always being placed over the edge of each abutment.

Fig. 56.

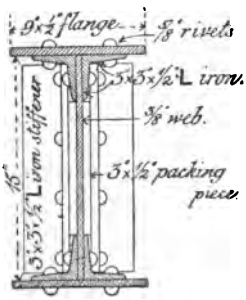


Fig. 55 shows an elevation, over the bearing, and Fig. 56 a section, made up from these data. It will be noticed that the rivets are spaced at

4-in. centres, as is usual in England; that the bearing is nearly $\frac{1}{12}$ of the span; and that the flange plates and L-irons are carried round the ends. The depth of 15 in. is that *between the flange plates*, or as nearly as possible that between the centres of gravity of the flanges, considering the L-irons to form part of these.

Having thus determined the form of the smaller girder, we will proceed to the larger. The depth of this, between the centres of gravity of the flanges, may be assumed as 27 in. The stress at the centre due to the distributed load of $1\frac{1}{4}$ tons per foot run is therefore, by the usual formula of $S = \frac{W\,l}{8\,d}$, equal to

$$\frac{40 \text{ tons} \times 32 \text{ ft.}}{8 \times 2{\cdot}25 \text{ ft.}} = \frac{1{,}280 \text{ ft.-tons}}{18 \text{ ft.}} = 71\tfrac{1}{9} \text{ tons.}$$

Set this up to scale over the centre of a line representing the girder (Fig. 57), and complete the parabolic outline of intermediate stresses.

Half the distributed load on the small girder is carried by the larger girder at the point C. This is equal to 10 tons, and the stress over the point C in either flange is found, as explained in Chapter IV., thus:—

$$S = \frac{\text{Reaction at A due to load at C} \times \text{distance A C}}{\text{depth}}$$

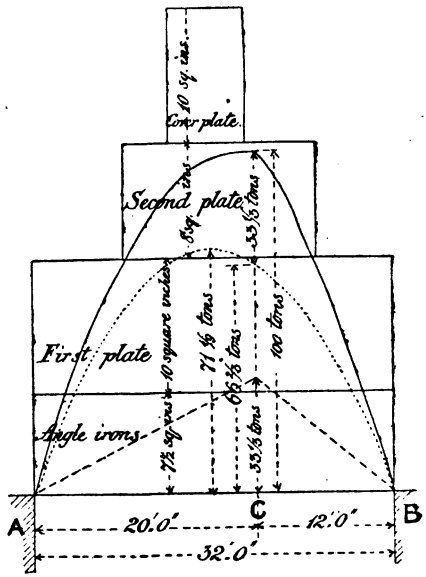
$$= \frac{3\frac{3}{4} \text{ tons} \times 20 \text{ ft.}}{2{\cdot}25 \text{ ft.}} = \frac{75 \text{ ft.-tons}}{2{\cdot}25 \text{ ft.}} = 33\tfrac{1}{3} \text{ tons.}$$

Set this up over C (Fig. 57) to the same scale as that used to set up the $71\frac{1}{9}$ tons over the centre, forming the triangular outline of intermediate stresses, as explained in Chapter IV. The triangular and parabolic outlines must next be summed, by adding together the ordinates to each

from all points of the line A B, thus producing a new outline of total stresses due to both the fixed and distributed loads, as shown by the firm line in Fig. 57.

The greatest stress in either flange is thus seen to be produced over the point C, where it is equal to $66\frac{2}{3}$ tons

Fig. 57.



(due to distributed load) + $33\frac{1}{3}$ tons (due to fixed load), making a total stress at C of 100 tons.

In the compression flange, therefore, at 4 tons to the square inch, an area of 25 in. of metal is required to meet the stress at the point C.

Of this, two 4 in. by 4 in. by $\frac{1}{2}$ in. L-irons would contain $7\frac{1}{2}$ sq. in., and the remaining $17\frac{1}{2}$ sq. in. would be more than contained by one 16 in. by $\frac{5}{8}$ in. plate, and one 16 in.

by $\frac{1}{2}$ in. As a matter of practice the thicker plate is usually placed next the L-irons.

The distance of 100 tons, scaled on Fig. 57, can now be divided proportionally to the areas of angle irons and plates, the first plate extending, with the angle irons, over the whole span, but the second plate being stopped, for the sake of economy, one rivet hole beyond the point where it overlaps the firm outline of total stresses.

The second plate is thus less than 20 ft. in length, and so can be made in one piece; but both the first plate and the angle irons have to be jointed, wrought iron not being rolled, as a rule, to a greater length than 20 ft., and the stresses which they bear have to be transferred over the joint by means of cover plates. By arranging the joints so as not to fall over one another, one cover plate can be made to suffice, provided that it is of the same section as the larger of the plates to be jointed.

The length of the cover depends upon the number of rivets required to convey the stress to it from the plate and back again on the other side; and for this purpose the tension flange should be considered, as theoretically the abutment on the joint in compression renders a cover unnecessary in the compression flange, though any slight error in the joint at once brings it into use. The rivets used would be somewhat more in diameter than the thickness of the thickest plate to be pierced; in this case $\frac{3}{4}$-in. rivets would be used. There would be four rows of these in a flange 16 in. wide, with a pitch of 4 in. from centre to centre. The effective sectional area in tension of the thicker plate is 16 in. $\times$ $\frac{5}{8}$ in. $-$ 4 ($\frac{3}{4}$ in. by $\frac{5}{8}$ in.) $= 8\frac{1}{8}$ in., which is capable, at 5 tons per inch, of resisting a tensile stress of $40\frac{5}{8}$ tons. This has to be conveyed through the shanks of the rivets to the cover, the rivets being in shear to the extent of $40\frac{5}{8}$ tons. Each $\frac{3}{4}$-in. rivet

contains ·44 sq. in. of metal, which, at 4 tons per square inch (the safe resistance of good rivet iron to shear), gives a resistance of 1·76 tons per rivet. The number of rivets necessary, therefore, to convey the tension from the plate to the cover is $\frac{40\frac{5}{8}}{1\frac{3}{4}}$, or $23\frac{1}{2}$ nearly. You cannot divide a rivet, so that 24 have to be used (see Fig. 58), making six rows of four each.

The consideration of the space between the joints is complicated by the fact that only two rivets in each row pass through the angle-irons to the cover, though the effective area for tension is four holes less than the total area—the rivets through the web having no effect in transferring stress to the cover. The effective area of the angle-irons is therefore $7\frac{1}{2}$ sq. in. $- 4$ ($\frac{5}{8}$ in. $\times$ $\frac{1}{2}$ in.) $= 6\frac{1}{4}$ in., which, at five tons per square inch, represents a stress of $31\frac{1}{4}$ tons. Taking ·44 in. as the area of a rivet, and its resistance to shear as $1\frac{3}{4}$ tons as before, this necessitates the use of 18 rivets, or nine rows of two. This would be exceedingly heavy riveting, necessitating the employment of an inordinately long cover plate. More economy can be effected by increasing the thickness of the cover from $\frac{5}{8}$ in. (which is that barely necessitated to convey the stress in the $\frac{5}{8}$-in. plate across its joint) to $\frac{7}{8}$ in. It then, after deducting rivet holes, contains 16 in. $\times$ $\frac{7}{8}$ in. $- 4 \times$ $\frac{3}{4}$ in. $\times$ $\frac{7}{8}$ in. $= 11\frac{3}{8}$ in., which, at 5 tons per square inch, would resist $56\frac{7}{8}$ tons stress. Of this there is $40\frac{5}{8}$ tons taken up to compensate for the cut plate, the balance of $16\frac{1}{4}$ tons, *plus* five rows of two rivets each (10 rivets) at 4 tons per square inch (which works out to $17\frac{1}{2}$ tons), making up $33\frac{3}{4}$ tons, which is more than the $31\frac{1}{4}$ tons stress in the cut angle irons. Thus the joint could not give by shearing of rivets between the joints and by tearing of cover and outer plate at the position of the joint in inner plate.

From the joint in the angle-irons to either end of the cover, however, the full complement of nine rows of rivets, as ascertained, would be required to avoid fracture by

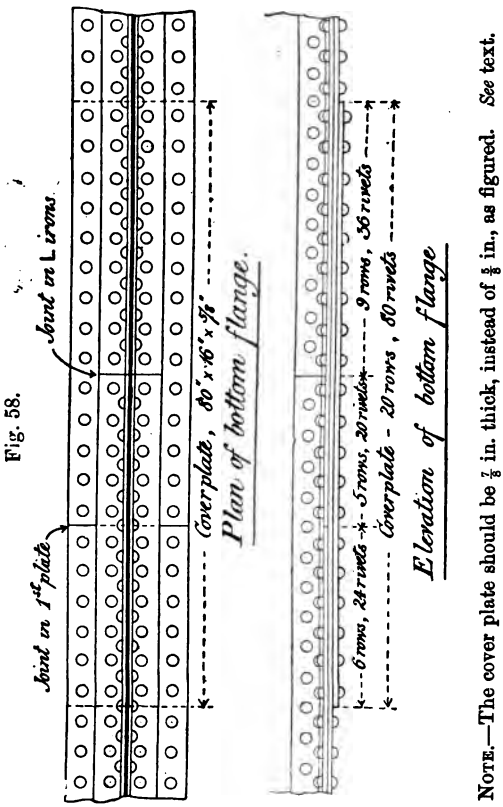

Joint in 1st plate
Joint in L irons
Coverplate, 30"x16"x⅝"
Plan of bottom flange.
6 rows, 24 rivets
5 rows, 20 rivets
9 rows, 36 rivets
Coverplate - 20 rows, 80 rivets
Elevation of bottom flange

Fig. 58.

NOTE.—The cover plate should be ⅞ in. thick, instead of ¾ in., as figured. *See* text.

shearing of rivets between angle-irons and plates, and tearing of plates at the last riveted section at end of cover plate.

The cover plate would thus contain a total of 20 rows of rivets at 4 in. pitch, and would be 80 in. in length. It would occur as near to the centre of the girder as circumstances would permit, its position being indicated on Fig. 57.

In larger girders it is often possible to make use of surplus metal lying outside the total outline of stresses (Fig. 57) to reduce either the area of cover plate or distance between joints, while in many cases a top plate can be slightly lengthened to serve the purposes of a cover by placing the joints near to its end. In fact, it is difficult to lay down any other rule for designing cover plates, save that every possible way in which fracture can occur should be considered. In the present instance, failure by bearing has not been entered into, as it is evidently safe in this respect.

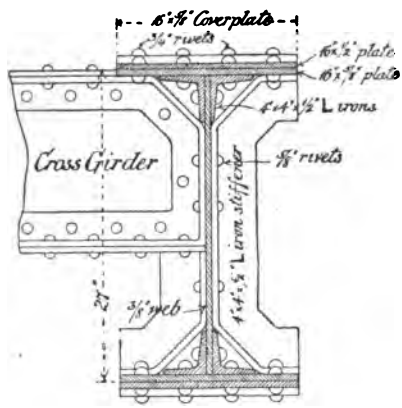
The greatest shear in the web is over the abutment B, where it is equal to 10 tons $\times \frac{5}{8} + \frac{40 \text{ tons}}{2} = 26\frac{1}{4}$ tons. At 4 tons per square inch this requires a sectional area of $6\frac{5}{8}$ in. to meet it, which is much more than contained by a web of 27 in. $\times \frac{3}{8}$ in.—the smallest obtainable, or at least of any value against corrosion.

The girder may now be considered complete, its section being as shown in Fig. 59, which also shows the connection with the cross girder, assuming that the top flanges are required to be on the same level. The cross girder, it will be noted, is connected to the web only of the main girder so as to avoid producing a torsional stress, and the connecting rivets should contain enough metal to resist, as a shear, the proportion of weight transferred from the one girder to the other, any needed beyond those shown connecting the angle-iron of cross girder to web of main girder being obtained by riveting an angle piece of $\frac{1}{2}$-in.

metal to the under side of lower flange of cross girder and to web of main girder, which, however, is not shown in Fig. 59.

The stiffeners can be arranged upon a somewhat better system than had to be adopted with the cross girder, it

Fig. 59.



NOTE.—The cover plate should be ⅞ in. thick, instead of ⅝ in., as figured. *See* text.

being now possible to bend them across the main angle-irons, and so to do away with the objectionable packing pieces. Where the web requires to be jointed, T instead of L-iron stiffeners are employed, the table of the T forming the cover plate.

CHAPTER VII.

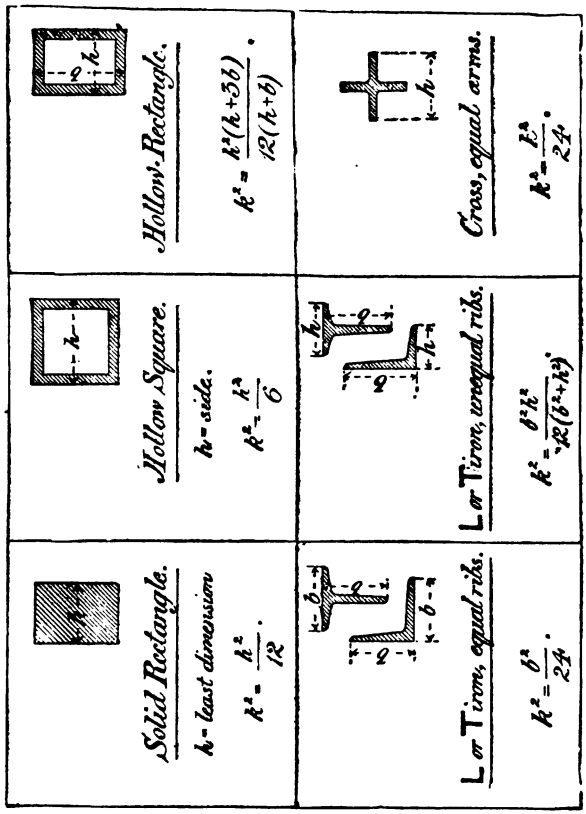
COLUMNS AND STRUTS.

COLUMNS and struts vary in their bearing capacity, not only with their sectional area, but also greatly with their length. This variation does not depend upon mathematical principles, and it has only been possible to arrive at a formula which will give an approximately accurate result after much patient investigation of the results of a long series of experiments. Several such formulæ have been put forward from time to time, but all depend upon certain definite hypotheses. The one given here is the well known "Gordon's Formula;" it is easy of application, and is as correct as any. It is for columns of less than thirty diameters in length, and is based upon the following assumptions—viz. (1) the ends must be planed or turned so that their surfaces will be perpendicular to the axis of the pillar; (2) the bases and caps must be planed or turned to fit the ends; (3) the pillars must be set with the axes vertical and directly under the centre of the load.

Where these rules cannot be complied with, as in the struts of a roof truss, a slight modification of the formula is necessary. Under ordinary circumstances, however, where the rules apply, it has been found that

$$\text{Breaking weight in tons} = \frac{F K}{1 + C\frac{l^2}{k^2}},$$

when K = breaking weight of the material in tons per inch, as ascertained in the usual way by crushing solid cylinders whose length is equal to twice their diameter



(in other words, K = 36 tons for cast iron and 16 tons for wrought iron), F = sectional area of metal, l = length in inches, C = 1/6,400 for cast iron and 1/36,000 for

wrought iron, while the value of k^2 in inches is, for various sections, that given in Figs. 60 to 69. Where one end is jointed or bears unevenly, use 2 C instead of C, and where both ends are jointed use 3 C.

Fig. 66.

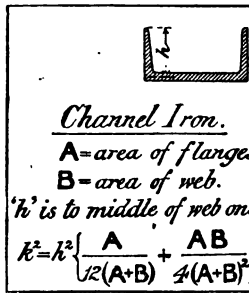


Fig. 67.

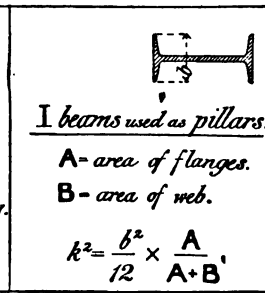


Fig. 68.

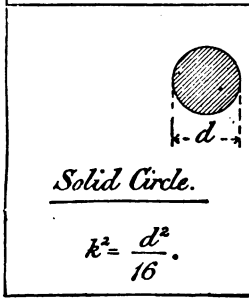
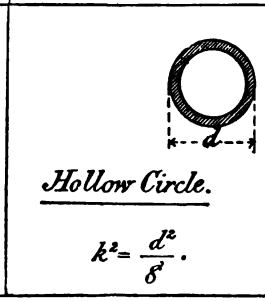


Fig. 69.



If it be required to ascertain the weight which the column will bear safely, the value of K must be one-sixth of the breaking weight in quiescent structures, and one-eighth of the breaking weight in structures subject to a moving load.

It is more often requisite to discover the necessary sectional area for a column which has to carry a given

load than to find out the bearing capability of a given column. In this case W, the load which it will be necessary for the columns to carry safely, is known; K must be taken as one-sixth or one-eighth, as explained above, of the breaking weight of the material per square inch, and the formula must read—

$$F = \frac{W\left(1 + C\frac{l^2}{k^2}\right)}{K}.$$

CHAPTER VIII.

FRAMED CANTILEVERS.

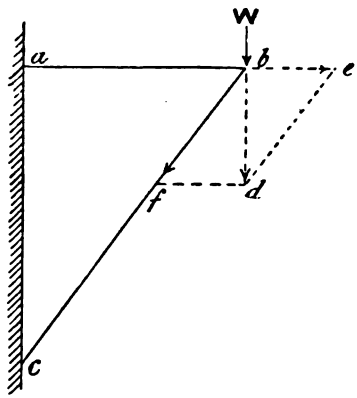
THE stresses in the different members of all framed structures can be ascertained by applying to them either the parallelogram or the polygon of forces, or both. The stresses discovered are those of tension and compression only, and it is assumed that they act in straight lines between the pins joining the different members together, and these lines alone are represented on the diagrams appended to this and the succeeding chapters.

The most simple form of framed structure is the framed cantilever, commonly known as the bracket or crane, according to the use to which it is put.

The stresses upon the members of a simple bracket (Fig. 70) are readily ascertained by means of the parallelogram of forces. Assuming the bracket to be loaded only at the end, set down vertically below the point of application of the load W a line *b d*, representing W to any scale of weights, the load acting downwards in the direction of the arrow. Round *b d* as a resultant form the parallelogram *b e d f*, with its sides parallel to the members *a b* and *b c* of the bracket; then *b e* will represent the stress in *a b*, and *b f* the stress in *b c*, to the same scale as that to which *b d* was set down. The direction of the stresses is shown by the arrows, *a b* being in tension on account of the pull of *b e* from the wall, and *b f* in compression, on account of the push of *b f* towards the wall.

The same result can be arrived at arithmetically with the help of Euclid I. 47, if the lengths of *a b*, *b c*, and *c a* be known (*a b c* and *b e d* being similar triangles), or by means of trigonometry, if either of the angles *a b c* or *b c a* be known. Given neat draughtsmanship, however, the graphic method of determining stresses is as accurate as either of the others, and its rapidity gives it a great advantage in practice. This advantage also can be

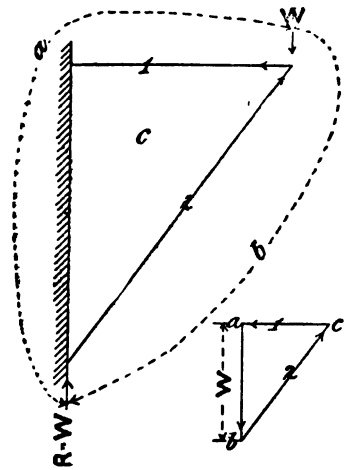
Fig. 70.



greatly increased, especially in complicated structures, by the employment of reciprocal diagrams. The application of one of these to the simple case now under consideration is shown in Fig. 71, in which, for simplification, Bow's notation of the spaces is employed. The reaction of the support is known to be equal to W (because the whole weight is borne by the support), and it acts upwards at the foot of the bracket. The *space* lying between R and W, over the bracket, is lettered *a*, that between W and R, under

the bracket, *b*. Then the space enclosed by the bracket must be lettered *c*. Great pains must be taken to grasp the fact that *spaces* are now being lettered on the sketch, and that these spaces correspond with similarly lettered *points* on the reciprocal diagram, in the following manner. Between the spaces *a* and *b* there lies the weight W acting vertically downwards. From the point *a* to the point *b* set to any scale of weights a distance vertically

Fig. 71.



downwards equal to W. Then between the spaces *b* and *a* there lies the reaction R acting vertically upwards, this being represented on the diagram by the line *b a*, drawn vertically upwards. A closed polygon, *a b*, *b a*, is thus formed in the one line between the points *a* and *b*, representing the external forces acting on the bracket, showing that these forces are in equilibrium.

In order to complete the diagram, and to ascertain the

position of the point to represent the space *c*, as well as to discover the stresses in the two members of the bracket, it is necessary to consider the forces acting at one of the joints. For the sake of convenience it is best to choose the joint at the apex of the bracket. Here, between the spaces *a* and *b*, there lies the load W, already represented by the line *a b*. Between the spaces *a* and *c* there lies the horizontal member of the bracket; from the point *a* in the diagram, therefore, draw *a c* horizontally and of unlimited length. Between the spaces *b* and *c* there lies the inclined member of the bracket; from the point *b* in the diagram draw *b c* parallel to this member till it cuts *a c* in *c*. Then, to the scale to which W was set down in the diagram, *a c* represents the stress in the member lying between the spaces *a* and *c*, and *b c* represents the stress in the member lying between the spaces *b* and *c*, the letters used reciprocating, and the triangle *a b c* in the diagram forming the triangle of forces about the apex of the bracket.

The stress in *d* is again divided at its foot into R, acting vertically upwards, and a thrust, equal to the stress upon the line 1, acting horizontally. This last has not been shown on the diagram for the sake of clearness.

Not only the value, but the nature also, of these stresses can be shown on the sketch. It is one of the properties of a triangle or polygon of forces of any number of sides, that if it be properly closed and represent in order the forces acting upon a point (such forces being in equilibrium), the direction of these forces will follow in consecutive order. Of the forces at the apex of the bracket, W is known to act vertically downwards from *a* to *b*. The other forces will therefore act, as shown by the arrows on the reciprocal diagram, from *b* to *c* and from *c* to *a*. If these arrows be transferred from the diagram to the

sketch, it will be seen that that on the inclined member pushes towards the joint under consideration, while that on the horizontal member pulls away from the joint—in other words, the latent force in the member between *b* and *c*, which meets the stress upon that member, pushes towards the joint, the member being in compression, while the member between *a* and *c* is in tension.

The above lengthy explanation of a simple principle should be most carefully studied and mastered, as upon it depends the unravelling of many a complicated problem.

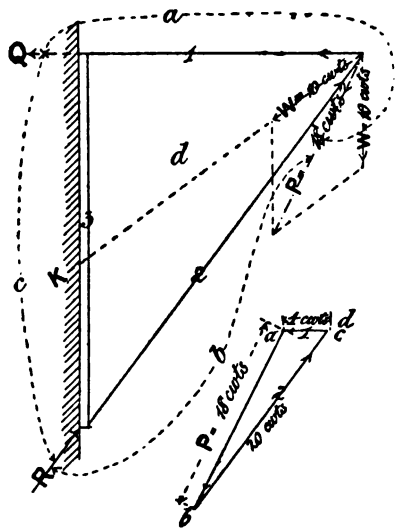
The stresses upon joints are of as great importance as are those upon the members of a structure, for the strength of the whole is only that of its weakest part. If, as is usual, the weight upon the bracket be carried at the apex as shown, but resting upon the extremity of the horizontal member, to which the inclined member is pinned, the shear on the pin (which generally exposes two shearing areas, each meeting half the shearing stress) is equal to the resultant of the load W and the tensional stress in the horizontal member—or in other words, is equal to the compressional stress in the inclined member.

Should the bracket carry a distributed load, half of this may be considered to be borne by the wall direct, and the remaining half to act as an end load, in which case the stresses due to this must be ascertained in the same manner as were those in the last example, substituting $\frac{W}{2}$ for W. Additional to the tensional stress thus found to exist in the horizontal member, however, it must be remembered that it acts as an ordinary beam carrying a distributed load between two supports (viz. the wall and the apex of the bracket), and that it must be proportioned accordingly.

When the cantilever is used as a crane—viz. when a

rope or chain passes over a pulley at the apex of the bracket to support a weight—it is not this weight only which goes to form the stresses in the members. The rope passing over the pulley pulls on it in two directions (see Figs. 72 and 73), the resultant P being the pressure bearing upon the pulley and causing the stresses. It is

Fig. 72.



this resultant, or the thrust in the member 2, whichever may be the greater, which has to be borne as a shear upon the pin at the apex joint.

The cranes shown in Figs. 72 and 73 are in the form of simple brackets, resting upon eyes let into the wall so that they may rotate freely to any required position. The chain (which is presumed to carry a weight of 10 cwt.) passes over the pulley and down to a winch at the point K.

The weight and the winch each pull upon the chain to the extent of 10 cwt., causing a resultant pressure upon the pulley P, which is equal to 18 cwt. in amount, and the direction of which is shown in the diagrams. In such a crane two cases are possible: it may be supported either by hanging on to the eye at its head, or by resting on the stirrup at its heel. In the latter case the diagram is a simple one, and is given in Fig. 72. We know at once that the external forces acting on the crane are the pressure P, the reaction R at the heel, equal in amount and direction to the stress on the member 2, and a direct resistance Q at the head, equal to the stress on the member 1. Common sense, a useful factor in the investigation of stresses as in many other matters, tells us that there is no stress in the member 3; and that the stress in 2 is conveyed in its full original force and direction to the wall by the eye at the foot acting as a very short cantilever—this eye being subject to a vertical end load and a horizontal thrust, respectively the vertical and horizontal components of the stress in 2, into which these can be again resolved on reaching the wall.

The space between Q and P on the sketch should be lettered *a*, that between P and R lettered *b*, and that between R and Q lettered *c*. The space in the centre of the bracket may be lettered *d*.

Between the spaces *a* and *b* there lies the pressure P of 18 cwt. Lay down, therefore, on the diagram from the point *a* to the point *b* a distance equal to 18 cwt., to a scale of weights to represent P and parallel to its direction. The other external force being as yet unknown, it is impossible to lay it down; but the stresses in the members 1 and 2 can be ascertained by drawing *a d* and *b d* (1 and 2) on the diagram respectively parallel to the members 1 and 2 lying between the spaces *a* and *d*, and *b* and *d*. The

stress on 1 is therefore represented by the line 1 on the diagram, which equals 4 cwt. to scale; and the stress on 2 is similarly shown by the line 2, and is equal to 20 cwt. The direction of these stresses is shown by the arrows, for P acting downwards, the arrow was set downwards from *a* to *b* on the diagram, and the other arrows on the diagram follow in rotation round the closed polygon from *b* to *d* and from *d* to *a*. These being transferred to the sketch show the arrow on 2 to point towards the joint under consideration, the member 2 being, therefore, in compression, while the arrow on 1 points from that joint, showing that it is in tension.

There being no stress in the member 3, it is evident that the spaces *d* and *c*, between which it lies, may be represented by one and the same point (lettered *d* and *c*) on the diagram. R, therefore, lying between *b* and *c*, is equal in amount and direction to *b c* on the diagram, the polygon of forces acting at the foot of the bracket being represented by the lines *d b*, *b c*, *c d*, which last is *nil;* and R may be divided up into vertical and horizontal resistances if required, as explained in the previous case. Similarly the polygon *c a*, *a d*, *d c* represents the polygon of forces about the head of the bracket, *c a* being opposite and equal to *a d*, and representing Q, the resistance of the wall at the head of the bracket.

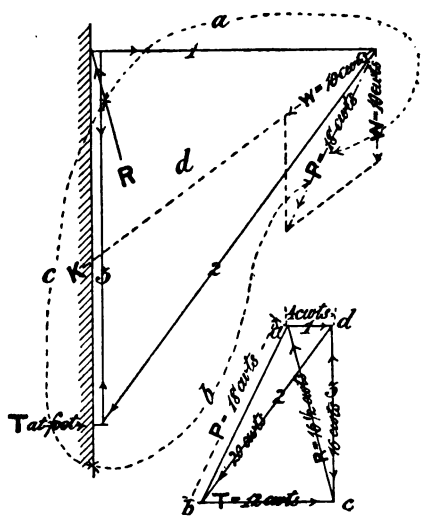
The case shown in Fig. 73, where the crane hangs at its head, requires somewhat further consideration. The external forces here, besides P, are a horizontal thrust, T, at the foot, and the reaction, R, at the head, of which neither the value nor direction is known, save that it is equal and opposite to the resultant of the stresses in the members 1 and 3.

The lettering may be similar to that of Fig. 72, the space between R and P being lettered *a*, that between P

and T being lettered *b*, and that between T and R being known as *c*, the space in the middle of the bracket being *d*.

From *a* to *b* on the diagram a distance may be set down as before, equal in amount and parallel in direction to P, and the triangle *a b d* may be completed, representing the pressure P and the stresses in 1 and 2. So far this is exactly similar to the previous diagram, the stresses, both

Fig. 73.



in amount and direction, in the members 1 and 2 being the same as those then ascertained. For the sake of clearness the arrows relating to this triangle are not again shown.

Both at the head and foot of the bracket, however, there are two, as yet, unknown forces. At the foot the directions of both (the thrust T, and the stress in 3) are known, which is not the case at the head, where the direction as

well of the value R remain unknown. By drawing *b c* on the diagram parallel to T, which lies between *b* and *c*, and by drawing *d c* parallel to 3, which lies between *d* and *c*, it is possible to fix the point *c* on the diagram to represent the space *c* on the sketch. The value of T is thus ascertained, by scaling *b c*, to be equal to 12 cwt., while the value of 3 is similarly found, by scaling *c d* on the diagram, to be 16 cwt., all the forces at the foot being represented by the triangle *b c d*.

The member 2 is known (from the consideration of the previous case) to be in compression, and therefore thrusts towards the joint at its foot, the arrow pointing downwards. This arrow is transferred to the diagram, pointing from *d* to *b*, the other arrows following round the triangle *d b c* in rotation from *b* to *c* and *c* to *d*. These being transferred back to the sketch, show T thrusting towards the joint, and 3 in tension pulling away from it.

At the head of the bracket there is, now that the value of 3 has been ascertained, but one unknown force, R; and this can be found, as it lies between the spaces *a* and *c*, by joining *a c* on the diagram. Its direction is parallel to *a c*, and its value 16½ cwt.

The forces acting at the head of the bracket are now represented by the sides of the triangle *a d c*. The member 3, lying between *d* and *c*, has just previously been found to be in tension, and therefore pulls away from the joint. This arrow, therefore, points downwards from *d* to *c* on the diagram (i.e. in the opposite direction to the arrow which denoted that the member 3 pulled away from the previously considered joint at the heel), and the other arrows, following in rotation from *c* to *a* and *a* to *d*, show, when transferred to the sketch, that R acts upwards, and that 1 pulls away from the joint and is in tension.

CHAPTER IX.

FRAMED BEAMS AND GIRDERS.

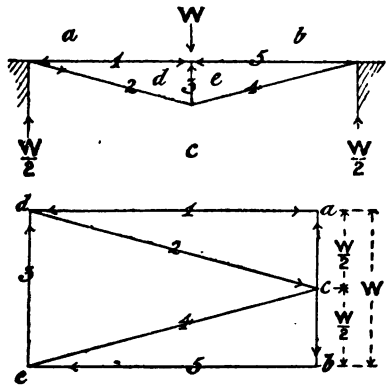
THE stresses on all framed structures can be ascertained mathematically by solving triangles, parallelograms, and polygons of forces; but as all these, round all the joints in a structure, can be included in one diagram of the kind used in the last chapter, and as such a diagram requires only accurate draughtsmanship for its solution in the place of a considerable acquaintance with mathematics, it is now almost universally employed; although on structures of magnitude and importance it is best to calculate the stresses by both methods, that the one may check the other. In this work, however, the diagram will be adhered to.

The simplest example of a framed beam is the trussed traveller (see Fig. 74). The greatest stresses upon the members are found when the weight which it has to carry is over its centre; for if the weight be in any other position a portion of it only would be carried by the strut in the centre (acting as an abutment), the remainder being carried directly by one of the main supports. With the weight in the centre, the reaction of each support will equal $\frac{W}{2}$. Letter the spaces as shown in Fig. 74, taking the spaces outside the structure first, and then those inside, in rotation as the sun moves. The space between the reaction of the left-hand abutment and W is thus

known as *a*, that between W and the other reaction as *b*, that between the two reactions as *c*, that between the members 1, 2, and 3 as *d*, and that between the members 3, 4, and 5 as *e*.

Between the spaces *a* and *b* there lies the load W, acting vertically downwards. Set to a scale of weights

Fig. 74.



a line vertically downwards from any point *a* to a point *b*, to represent W. Between the spaces *b* and *c*, $\frac{W}{2}$ acts vertically upwards. Set up vertically from *b* to *c* a distance to represent $\frac{W}{2}$; similarly set up from *c* to *a* a distance to represent $\frac{W}{2}$ acting upwards between the spaces *c* and *a*. The line *a b c a* on the diagram now forms a closed polygon, which represents the forces W, $\frac{W}{2}$ and $\frac{W}{2}$ acting externally upon the structure, and which are themselves in equilibrium.

In order to ascertain the stresses in the members, it is necessary to form, on the diagram, polygons of the forces which act at each of the joints in succession, and it is usual to commence with the joint over the left-hand abutment. Here, besides the reaction $\frac{W}{2}$ upwards between c and a, there act the stresses in the members 1 and 2. As the line 1 lies between the spaces a and d, draw a line parallel to it on the diagram from a till it is cut by another from c drawn parallel to the line 2, which lies between c and d, these new lines on the diagram meeting at the point d. Then $a\,d$ on the diagram represents to scale the stress on the member lying between the spaces a and d, and $c\,d$ represents the stress upon the member lying between c and d; the lines $\frac{W}{2}$, 1, and 2 forming a close polygon in the diagram. $\frac{W}{2}$ is known to act upwards, and the other forces, following in rotation, necessarily act as shown by the arrows from a to d and from d to c. These arrows, on being transferred to the sketch, show 1 pushing towards and 2 pulling from the joint—in other words, 1 is in compression and 2 in tension.

Hitherto three forces only have acted on any one point; but directly under the load four forces will be found—viz., W and the stresses 1, 3 and 5. The values of two of these only are unknown, and the others can therefore be found, their directions being known, and it being possible to close the polygon about the joint by drawing $d\,e$ on the diagram parallel to the line 3 lying between d and e on the sketch, and meeting it by $b\,e$ drawn parallel to the line 5 lying between b and e on the sketch. The line 1 is known to be in compression, and therefore acts towards this joint, just as it was shown to do towards the joint at

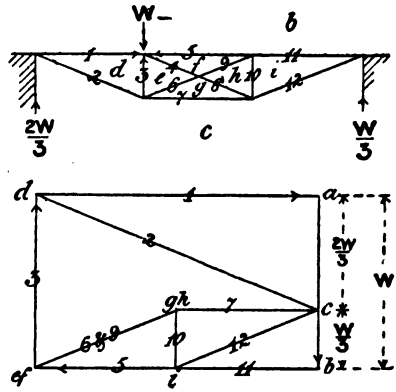
its other extremity, and its direction may therefore be represented by an arrow from *d* to *a* on the diagram. W acts downwards from *a* to *b*, maintaining the rotation, which is kept up by 5 acting from *b* to *e*, and 3 acting from *e* to *d*. This closes the polygon, 1, W, 5 and 3 on the diagram, representing the stresses on 1, W, 5 and 3 respectively in the sketch, and the direction arrows, when transferred, show that 5 and 3 are both in compression, acting towards the joint. It will be noticed that the stress in 3, which is directly under, and has its line of action coincident with that of the load, is equal to W. The stress in 4 can be ascertained by joining *c e* on the diagram, and if the working has been correct *c e* should be parallel to the line 4 (which lies between the spaces *c* and *e*) on the sketch. The direction in which it acts can be found by following the arrows round either of the polygons of forces of which *c e* forms a part—viz. either of those representing the forces acting at one of the extremities of the line 4. These arrows have not been shown, to avoid confusion, 4 being in tension.

A larger trussed traveller, with two struts and cross braces, is shown in Fig. 75. The greatest stresses upon the members are evidently produced when the weight is directly over one of the struts; but it must be carefully borne in mind in designing such a structure that the weight, which is a travelling weight, will sometimes be over one strut and sometimes over the other, while the members 1, 5 and 11 must be made sufficiently strong to carry it as a load placed centrally upon either of them.

The solution is very similar to that of the previous example until the joint under the load is reached, save that the reactions are not equal. The stresses in the members 1 and 2 can be ascertained as previously explained; but at the joint under the load the forces which

act are five in number—viz. the load W, and the stresses in the members 5, 4, 3 and 1. Of these, three are unknown, so that without an assumption the polygon cannot be completed, nor can the stresses upon these members be determined. By calling in the assistance of common sense, however, in this, as in most cases, an assumption is rendered possible which will permit of the required solution being arrived at. Bearing in mind what occurred during the solution of the previous example, we may fairly assume

Fig. 75.



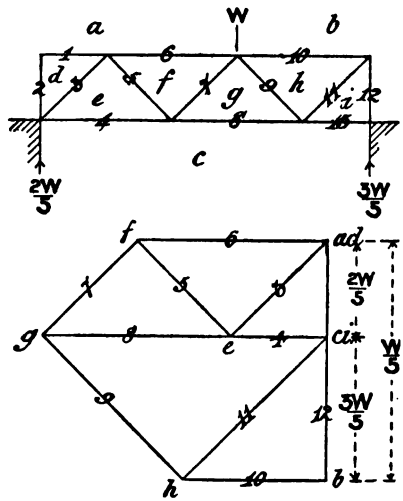
here that the member 3, being directly under and coincident in direction with the weight, is placed in compression to the extent of that weight. Set down on the diagram, therefore, a vertical line *d e* to represent the stress in the member 3, lying between the spaces *d* and *e*, equal in value to W. The member 1 being in compression acts towards the joint, this being represented by the arrow from *d* to *a* on the diagram. The member 3 we know to be also in compression, so that it acts upwards towards the joint; and, therefore, the distance *d e* must be set down-

wards so that the arrow may act upwards from *e* to *d* to preserve the continuity of direction of the arrows from *e* to *d*, and *d* to *a*. The point *e* is thus absolutely fixed, and it becomes possible to complete the polygon of forces by drawing *e f* from *e* and *b f* from *b*, parallel to the lines 4 and 5 (lying between the spaces *e* and *f* and *b* and *f*, respectively) till they meet in the point *f*—which in this instance occurs at *e*, this being from the construction in a horizontal line with *b*, thus showing that there is no stress upon the member 4. The direction arrows pertaining to the polygon *a b f e d a* are the only ones shown on this diagram, to avoid confusion, and these show that all the members attached to the joint under the load are in compression. The completion of the diagram is now easy upon the lines already explained, there being no stress on the member 8—the only polygon in any way out of the common being *b i h f b*, representing the stresses in the members 11, 10, 9 and 5, where *b i* and *f b* lie in the same straight line but in opposite directions.

Having so far cleared the way, the consideration of framed girders of any ordinary type should become a comparatively easy matter, and no difficulty whatever should be found in determining the stresses upon the members of the Warren girder shown in Fig. 76, so long as it is subject to only one load at any point. With the load as shown there would be no stress upon either of the members 1, 2 and 13, while the member 12 would be in compression to the extent of the reaction of the abutment under it. The only polygon, the arrows pertaining to which are shown, is *a b h g f*, representing the forces acting at the joint under the load; but all the other force polygons can be readily followed, the members 6, 10, 3, 7, 9 and 12 being in compression, and all the others which are subject to any stress at all in tension.

Girders of the form shown in Fig. 76 are not often employed, but they are occasionally of great value, especially when a bridge has to be thrown where there are two good natural abutments at different levels, the members 12 and 13 being dispensed with, 10 and 11 hanging on the abutment, as shown in Fig. 77, which is shown carrying a distributed load on the top flange. The

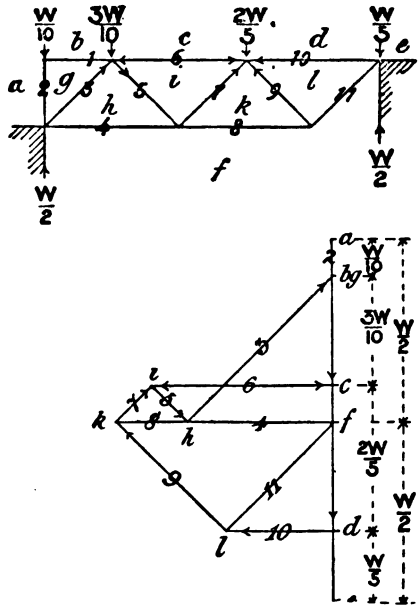
Fig. 76.



members 1, 6 and 10 act as a continuous girder, although, as a rule, in practice they are calculated as three separate supported girders, each carrying a distributed load, in addition to the stresses brought upon it as part of the framed structure. In either case, half of the load on 1 is carried by each of its abutments, and so with 6 and 10, making the loads on these supports $\frac{W}{10}$, $\frac{3W}{10}$, $\frac{2W}{5}$, and $\frac{W}{5}$

respectively as shown, in proportions of the total load W. The load being distributed, the reaction of each abutment is $\frac{W}{2}$, and the setting-down of the polygon of external forces is easy, $\frac{W}{10}$, $\frac{3W}{10}$, $\frac{2W}{5}$, and $\frac{W}{5}$ acting downwards,

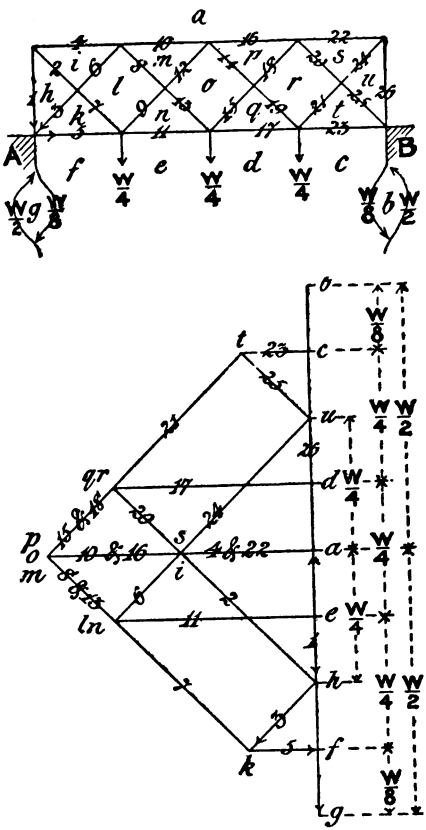
Fig. 77.



and each reaction of $\frac{W}{2}$ acting upwards, forming the closed polygon *a b c d e f a*. The polygon to represent $\frac{W}{10}$ and the stress upon the members 1 and 2 is readily found to be *a b g*—all lying in the one line *a b* on account of *b*

and g coinciding, there being no stress upon 1. Everything else is found as usual; but the polygon $beihg$,

Fig. 78.



which represents the forces acting at the joint under $\frac{3W}{10}$, is worth notice, on account of the crossing of its boundary lines.

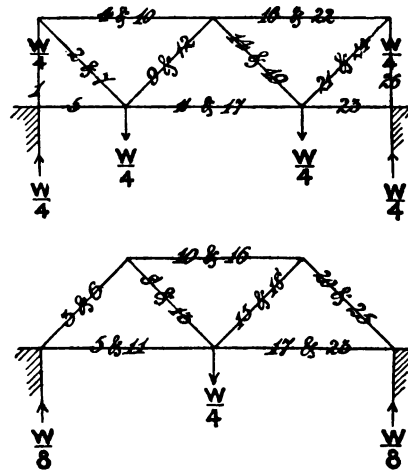
In the previously considered cases it would have been possible by inspection to discriminate between the members in compression and those in tension; but in such instances as that given in Fig. 77 this is by no means easy, and the arrows should be consulted. They are shown for the polygons of the forces acting about the joints under $\frac{3W}{10}$ and $\frac{2W}{5}$, and denote that the members 2, 6, 10, 3, 7 and 9 are in compression, while all the rest are in tension, except 1, which bears no other stress than that which comes upon it as a beam bearing the distributed load of $\frac{W}{5}$ to its two supports.

Another common form of framed girder is that shown in Fig. 78, generally known as the "trellis girder." It is assumed in Fig. 78 that the load is distributed and lies entirely upon the bottom flange, as is very commonly the case; and fresh difficulties are met with at once. It is found that at each abutment there act two forces: a small proportion of the load, in this case $\frac{W}{8}$, acting vertically downwards, and the reaction $\frac{W}{2}$ acting vertically upwards. In order to enable the space between these two forces to be lettered, their direction lines have to be looped, as shown. This done, the lettering can be proceeded with as usual, and the polygon of external forces can be readily laid down, commencing with $\frac{W}{2}$ acting upwards from *a* to *b*, to represent the reaction which acts upwards between the spaces *a* and *b*, proceeding with $\frac{W}{8}$ downwards from *b* to *c*, $\frac{W}{4}$ downwards from *c* to *d*, and so on, and closing

with $\frac{W}{2}$ upwards from g to a, to represent the other reaction between the spaces g and a. At the abutment A, the two external forces, $\frac{W}{8}$ downwards and $\frac{W}{2}$ upwards, are now known; but the three other forces, viz. the stresses upon the three members 1, 3, 5, are all unknown. It is therefore impossible to complete a polygon of the forces acting at the abutment A, without arriving first at the stress on one of these members by logical reasoning. The readiest way to do this is to dissect the trellis girder which we are considering into the two Warren girders of which it is made up (see Fig. 79), disregarding the small proportions of the load which lie directly on the abutments, and which, consequently, do not go to cause stresses upon the members of the structure. It will be seen at once, by inspection, that the members 1 and 26 are strained only to the extent of $\frac{W}{4}$, and that they are in compression. The stresses in the whole girder may be solved by solving those in the members of these component girders, adding similar and subtracting dissimilar stresses on the same member wherever they occur; but it is much easier to combine the diagrams in one, as shown in Fig. 78. A little practice will make it possible to dispense with the dissection shown in Fig. 79, and to determine the stresses in the members 1 and 26 by means of a mental dissection of the parts only. Having thus determined the stresses in these members, the rest is easy; $\frac{W}{8}$ acting downwards from f to g, and $\frac{W}{2}$ upwards from g to a. $\frac{W}{4}$ (which represents the compressional stress in the

member lying between the spaces *a* and *h*, and which member, therefore, presses towards the joint), must act downwards from *a* to *h*, to preserve the continuity of direction of the lines in the force polygon. This fixes the point *h*, and the lines *h k*, *k f* follow as a matter of course, determining the stresses upon the members 3 and 5, and completing the polygon *f g a h k*. The direction arrows

Fig. 79.

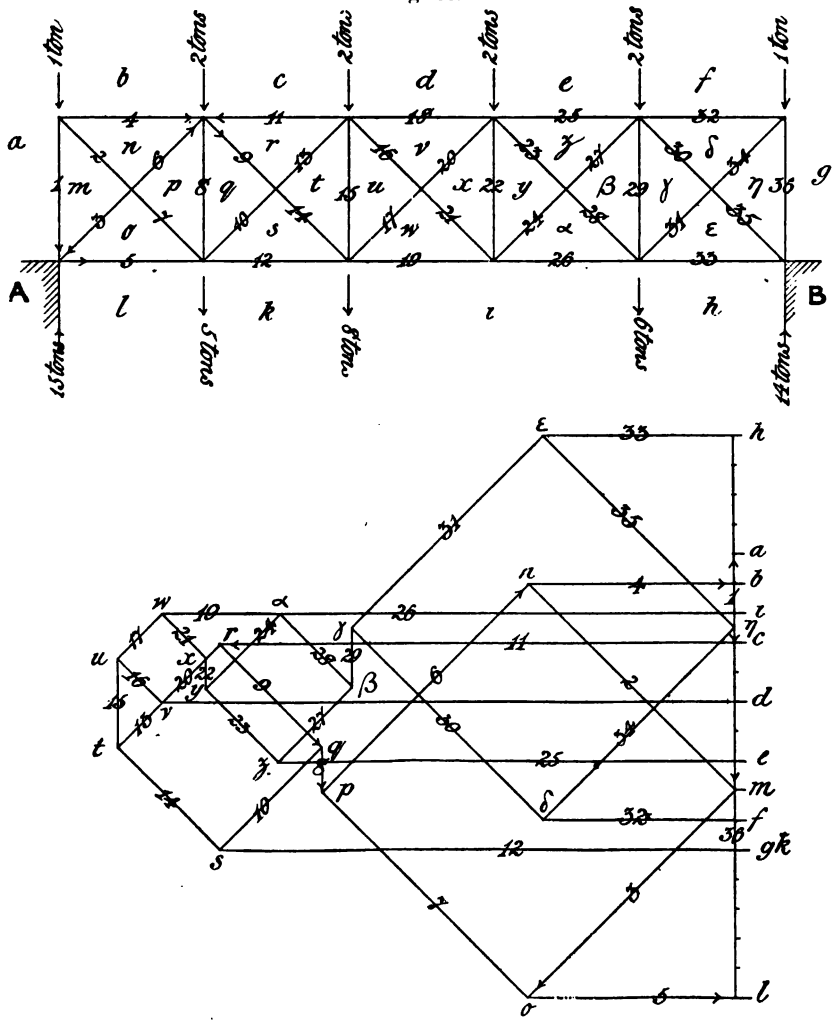


referring to this polygon are the only ones shown; but all others can be ascertained as usual, the members 1, 4, 10, 16, 22, 26, 3, 6, 20 and 25 being in compression, and all the rest in tension except 9, 12, 14 and 19, in which equal and opposite stresses meet and neutralise. Theoretically these members might be omitted, but practically it would be unwise to leave them out, as any accidental or temporary inequality of the loading would bring them

into use at once, and in any case they are of value in stiffening the structure.

Lattice girders are, perhaps, in more common use than either Warren or trellis girders, and present difficulties of their own to the solution of the stresses. An example is given in Fig. 80, loaded, as it might be, with an evenly distributed wall of half a ton per foot run on the top flange, and by loads of five, eight and six tons respectively at different points on the bottom flange; these loads being brought upon it by cross girders. The diagram produced is a complicated one; but it only requires care to enable anyone to follow it through. The external loads are set vertically downwards and upwards in succession as usual, till the polygon *a b c d e f g h i k l* is complete, the reactions having been ascertained either arithmetically or by means of the polar diagram. Of the forces at the abutment A, only the reaction (15 tons) is known, the stresses in the three members 1, 3 and 5 being unknown. It requires but little consideration, however, to determine the stress in 1. Of the reaction of 15 tons, 1 ton is met at once by the load on the top flange over the support. The member 1 is therefore put into direct compression to the extent of this one ton. Now it may be fairly assumed, as the girder is symmetrically framed, that each of the loads acts half at the top and half at the bottom of the vertical member to which it is attached. Similarly, of the remaining 14 tons of the reaction at A, it may be assumed that 7 tons acts at the foot of the girder, and that the other 7 tons is conveyed by the member 1 to the top flange, putting this member in compression to the extent of 7 tons. It has also been found to be in compression to the extent of 1 ton, which it directly conveys, so that it is totally compressed to the extent of 8 tons. This compression acts towards the joint, and as the reaction between

Fig. 80.


1 ton
2 tons
2 tons
2 tons
2 tons
1 ton
a
b
c
d
e
f
g
A
B
l
k
i
h
1½ tons
5 tons
8 tons
6 tons
14 tons

l and *a* acts upwards, in order to preserve the continuity of direction of the lines of the polygon of the forces about the abutment, the line *l a* must be set vertically upwards and the line *a m* vertically downwards. The polygon *a m o l*, to represent the reaction of 15 tons and the stresses in the members 1, 3 and 5, is now readily completed, the arrows showing that 3 is in compression and 5 in tension. The diagram can be easily continued till points have been found to represent the spaces *n*, *p* and *o*, by which time it will have been discovered that the members 4 and 6 are in compression, and 2 and 7 in tension. At the head of the member 8, however, three unknown forces are met with in the stresses on 8, 9 and 11. The load at the head of 8 is, however, half carried by the framing at its head and half by that at its foot—in other words, the member 8 conveys one ton of the two tons resting on its head to its foot, and so is put into compression to the extent of one ton. Similarly, in conveying to its head half of the load of 5 tons hanging from its foot, it is put into tension to the extent of $2\frac{1}{2}$ tons. The total stress, therefore, to which the member 8 is subject is 1 ton $- 2\frac{1}{2}$ tons; or $- 1\frac{1}{2}$ tons (compression and tension being respectively designated by the algebraical signs + and −, as is often done for convenience).

Similar reasoning will give the stress upon the member 15 as − 3 tons, that in 22 as + 1 ton, and that in 29 as − 2 tons. These being found, the remainder is easy work. The arrows of direction referring to the members about the head of 8 are shown; but, to avoid confusion, the remainder are omitted: 1, 4, 11, 18, 25, 32, 36, 3, 6, 10, 13, 16, 21, 23, 28, 30, 35 and 22 being in compression, and all the other members in tension.

It must be carefully noted that the assumption that half of each of the supported loads is carried at the head

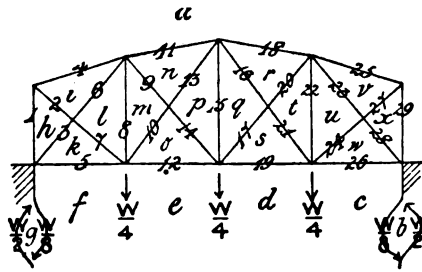
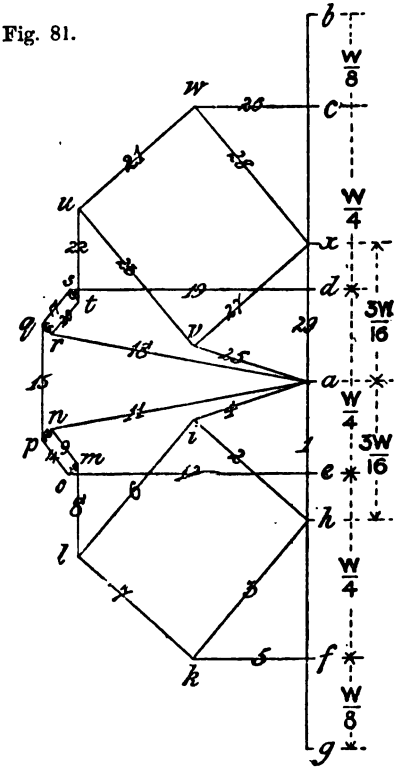

a
f
W/4
e
W/4
d
W/4
c
g
W/8
b
W/8

Fig. 81.


b
W/8
w
c
W/4
u
x
d
3W/16
q
t
v
a
W/4
n
p
m
i
3W/16
e
o
h
W/4
l
k
f
W/8
g

and half at the foot of the vertical member to which it is attached can only be made when the construction is sufficiently good for the joints to be taken to be perfect. This may generally be considered to be the case in iron structures, provided that they are put together under efficient supervision.

This assumption also fails if the structure be not symmetrically framed at the heads and feet of the vertical members, as in the bow-string lattice girder shown in Fig. 81, although by that assumption a fair approximation to the truth can be arrived at. By studying the diagram it will be seen that the stresses in the members 1, 8, 22 and 29 have been thus assumed, with the result of discovering the stress upon the member 15 to be one-sixth more than half the load carried at its foot. Had the upper flange been more nearly horizontal, the error would have been less; but the stress upon some of the struts would not have been lessened to so great an extent. The problem is one of a class known as indeterminate—i. e. impossible of accurate determination; but for all that, girders of this form are frequently employed, it being possible to arrive at the stresses upon the members with a sufficient approximation to accuracy for practical purposes, while the æsthetic effect is good; and the stress upon some of the principal struts is less than would be the case were the mean depth maintained throughout. The nearest approximation to the correct stresses is perhaps obtained by making separate diagrams for each system of braces, and averaging the results; but absolute correctness cannot be secured.

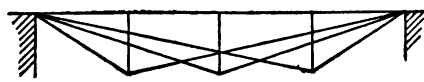
A form used to a considerable extent in bridges abroad, where timber for the struts and hemp or wire ropes for the ties have to be relied on, is shown in Fig. 82. Several modifications are possible; but in any case, to ascertain

the stresses on the members, it is necessary to dissect the framing, treating each simple truss as shown in the example of a trussed traveller given in Fig. 74.

For the common suspension girder another assumption is necessary before the stresses can be ascertained; and the determining of the stresses and the design of the girder must go hand in hand. It is evident (see Fig. 83) that the members 4, 6, 8 and 10 are each in tension to the full extent of the loads suspended from their feet, while the loads resting on the abutments have no effect upon the structure whatever.

In order to proceed, it is necessary either to assume that the central member 7 is horizontal, or to assume the inclination of the two extreme members 3 and 11. It is

Fig. 82.

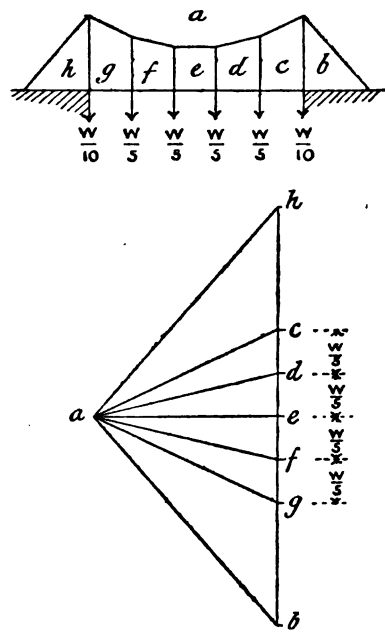


also necessary that the stress in 7 should be assumed as well as its direction, if that course be chosen, as its direction alone will not fix the point *a* in the diagram. Join *a c*, *a d*, *a e*, *a f* and *a g*, and draw the members which are not yet determined parallel to their representative lines respectively. By this means both the stresses upon the suspending chain and also its curve are determined, and the completion of the diagram gives the stresses on the vertical supports 2 and 12 and on the ties 1 and 13, which last have to be anchored down by heavy weights, sufficient in amount to resist their pull.

The application of what has gone before to other cases, which may vary to an infinite extent, must now be left to the student's ingenuity. Once the principle is

thoroughly grasped, no problem should prove too difficult of accomplishment, save the indeterminate cases, when

Fig. 83.



approximate results only can be hoped for. Travelling loads do not come, as a rule, within the province of the architect to meet, and will not be considered in this work.

CHAPTER X.

ROOFS—DEAD LOAD ONLY.

ALTHOUGH in roofs of large span and in exposed situations it is a matter of necessity that provision should be made to resist the pressure of the wind, yet, in the ordinary cases which occur in the practice of every architect, it is more customary to design the members to resist a vertical dead load only, a sufficient allowance being made to cover not only the weight of the material employed, but that of a considerable covering of snow also, with a slight addition to allow of ordinary wind pressures being sustained. It is thus a common thing to calculate for a total vertical dead load of 60 lb. per square foot, this being sufficient, under normal conditions, to fulfil all these requirements.

Such a load, spread over the whole surface of a roof, is conveyed by the common rafters to the purlins, and by them to particular points on the principal rafters. The common rafters thus have small loads to convey across short spans—from purlin to purlin, and though in large roofs they have to be calculated as beams bearing distributed loads, usually their sizes are determined by "rule of thumb," simply because there are practical difficulties in the way of procuring and securing rafters, whether of iron or wood, which contain only the amount of material theoretically necessary. Purlins, however, should always be calculated, and even in roofs of moderate span they often have to be trussed to enable them to convey the

loads brought upon them without their being made of inconvenient depth.

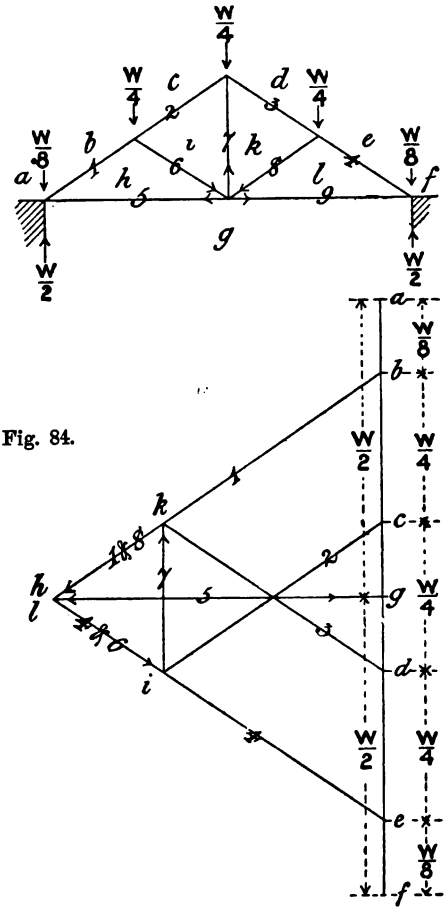


Fig. 84.

The main trusses of all roofs should be calculated, yet it is a common practice for architects to depend upon tables

of scantlings, to be found in various textbooks, and upon their own ideas of what ought to suffice. There is some

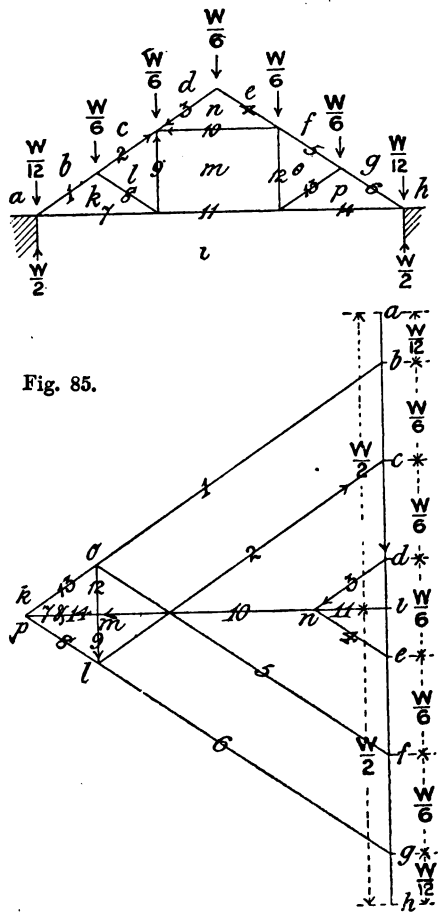


Fig. 85.

excuse for this where wood is the material employed, as timbers which would be of sufficient scantling to resist the

direct stresses brought upon them, would be insufficient for the formation of good joints at their extremities. The

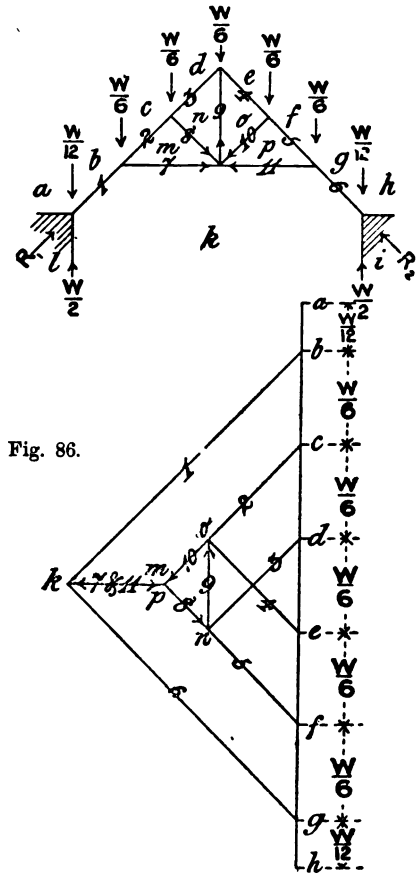
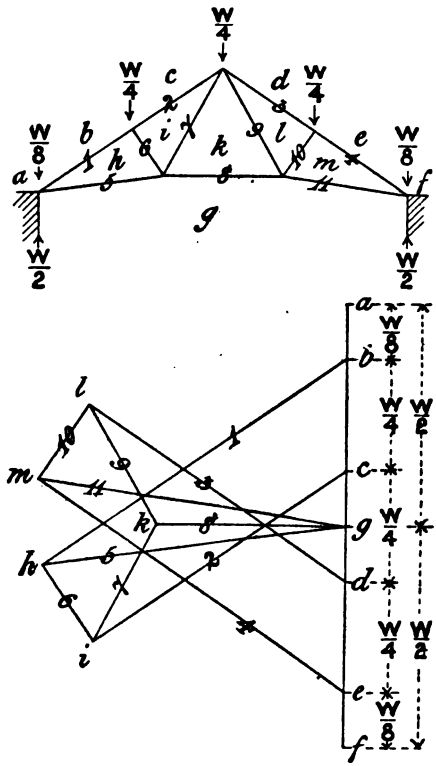


Fig. 86.

amount of timber required at the joint thus fixes the scantlings throughout, for it would be the merest waste

of labour and material, unless necessary for æsthetic effect, to cut away from the remainder of the member till only

Fig. 87.



the theoretically necessary amount of timber were left to resist the direct stress. With iron, a comparatively expensive material, and easily forged to any required

form, it is a different matter. The stress upon each member and every joint, according to the circumstances of each particular case, should be accurately determined, and sufficient metal—no more—should be provided to meet it.

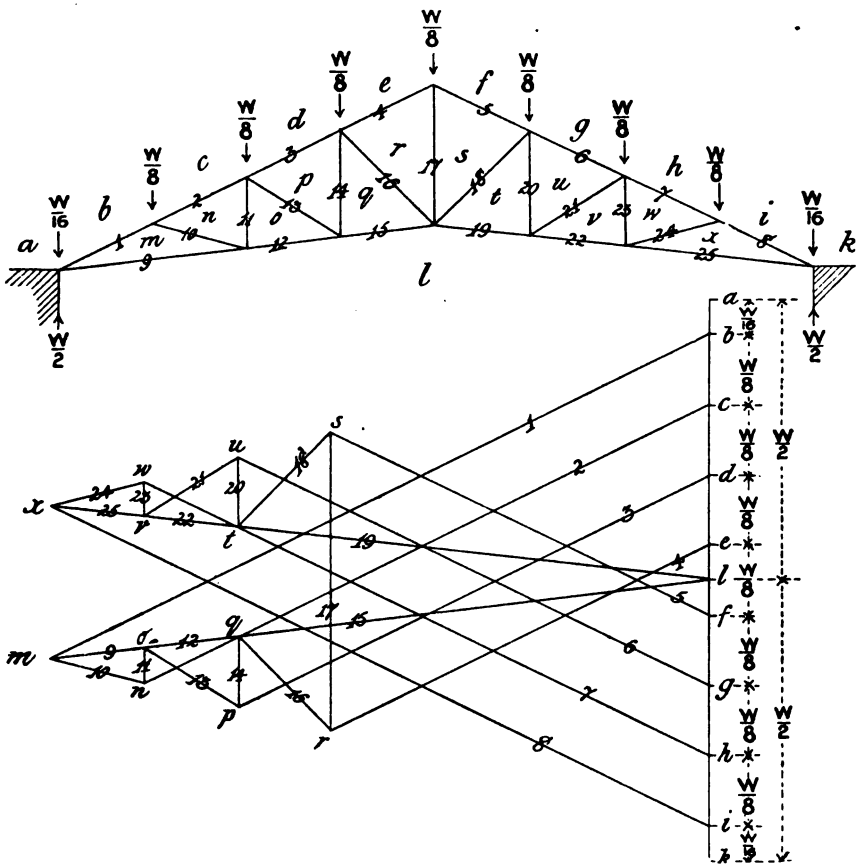
Few cases present any difficulty so far as the determination of the stresses in the members is concerned. Figs. 84 and 85 illustrate the ordinary king and queen-post roofs. No explanation of the diagram is necessary to those who have read the previous chapter; but it may be pointed out that in the member 11, Fig. 85, the tensional stress *m i*, as shown on the diagram, is that which would be brought upon it if no straining piece were used. The introduction of a straining piece would cause the tension in the tie-beam to remain uniformly equal to *k i* thoughout, while the straining piece itself would be placed in compression to the extent of *k m*. The direction-arrows are only shown round the foot of the king-post in Fig. 84, and the head of the queen-post 9 in Fig. 85. The horizontal distance between the points of application of the purlins is assumed to be uniform in each case, the proportions of the total load acting upon each joint being accordingly uniform also. If it be necessary in any particular case to space the purlins unequally, care must still be taken that the joints shall be vertically under the point of application of the purlins, and the proportions of the loads falling over each joint should be carefully ascertained. If the purlins do not rest directly over the joints, great inequalities of loading are the result, and the principal rafters are immediately put into the position of beams carrying fixed loads which they distribute to the joints.

The case shown in Fig. 86 requires somewhat further consideration. The load lying over the abutment is directly resisted vertically by the abutment; but the thrust on the

lower part of the principal rafter, not being counteracted by a tie-beam, is met by a reaction equal to it in amount and opposite in direction. The readiest method of ascertaining this is to consider the truss as ceasing with the collar-beam, the reactions at the feet being in the directions of the principals. By drawing *g k* parallel to 6 and *b k* parallel to 1 till they meet in *k*, the polygon of external forces *b c d e f g k* can be completed, giving *g k* as the reaction or thrust along the principal rafter 6, and *b k* as that along 1, to the scale of weights employed. The total thrust on the abutment is equal to the resultant of *b k* or *g k* and $\frac{W}{6}$, which has to be met by the weight of the wall, or, if this be of itself insufficient, by a buttress.

Where this form of roof is employed there is often a curved rib (solid) attached to underside of the collar and principals for the sake of appearance, and it is very commonly supposed that if such a rib be employed it will in some manner act as a tie and obviate the necessity for a buttress. This, however, is an entire mistake. This rib may (or may not, for even this depends upon the construction, and the rafter is usually strong enough without additional help) add to the stiffness of the rafter; but it cannot change the direction or lessen the amount of the thrust against the top of the wall until such thrust becomes active by the spreading out of the walls, and then only if it be very carefully jointed. It will be noticed from the direction-arrows that the collar, which is commonly thought to act as a tie, and to serve to a small degree the purpose of a tie-beam at the foot, is actually in compression. It prevents sagging of the rafters under the weight brought upon them, but does not tie in their feet unless the rafters be so strong as to carry the weight acting at the junction of collar and rafter without assistance, in which case an entirely different diagram

Fig. 88.



would be necessary, and the respective loads would have to be considered as acting at the head and feet of the truss and over the struts only.

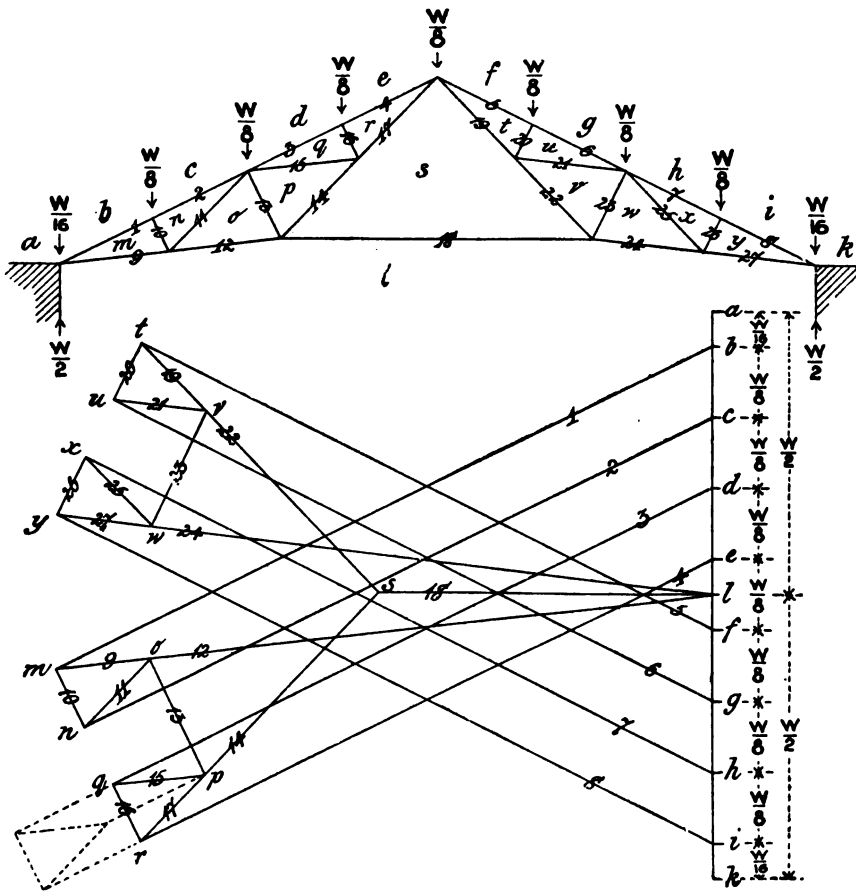
It is to be noticed that in king-post trusses there is no theoretical necessity for the king-post to be attached to the tie-beam at all, the only advantages gained by making this attachment being the prevention of sagging in the tie-beam from its own weight—until wind force is taken into consideration, when the value of the tie and king-post being joined is immediately felt.

The ordinary form of iron roofs shown in Figs. 87 and 88, with their many varieties, require little consideration, the stress diagrams being readily ascertained; but it is to be noted that the cambering of the tie-rod, which is customary, while it reduces the length of the struts, adds considerably to the stresses upon both the rafters and ties, and is thus by no means an unmixed blessing. The better forms of iron roof are therefore clearly those of the same type as that shown in Fig. 87, which, without adding greatly to the stresses upon the rafters, reduces the length of the struts to a minimum.

This type of truss can be adapted for a large span, as shown in Fig. 89, the determination of the stresses being still fairly simple, in spite of three unknown stresses being met with when the strut 13 is reached, whether the members at its head or its foot be considered. These can be found by making a trial diagram (dotted in Fig. 89) to ascertain the stresses on the members 16 and 15. Whatever may be the amount of stress on the rafters 3 and 4, it is clear that $q\ r$ will denote, both in amount and direction, the stress on the strut 16, and by drawing lines from its extremities q and r parallel to 15 and 14 respectively, a point is found at their junction which will represent the position of p as compared with q and r. It is then only necessary to slide the tri

angle *q p r* up the lines *q d*, *q e*, maintaining the inclinations of its three sides till the line *o p* is met by the apex *p*, to determine the length of *o p* (which scales off the stress upon the member 13), and so to reduce the number of unknown forces about either the head or foot of 13 to two, thus enabling the diagram to be completed.

Fig. 89.



CHAPTER XI.

ROOFS—WIND PRESSURES AND COMBINED LOADS.

THE only live load to which roofs are to any extent exposed under ordinary circumstances is the pressure of the wind; but this is of such importance, and may be considered in so many different ways, as to demand a chapter to itself. The different conditions which may arise, with the varying methods of treatment which they necessitate, are easiest compared by retaining the same type of roof for each illustration, and a very common and useful form of iron roof (see Figs. 90 to 94) has been chosen for the purpose, the same system of investigation as that employed being, however, applicable to all types alike.

The most natural assumption is that the wind acts horizontally against one side of the roof, though this in reality does not often occur. Each side of the roof is exposed in its turn to the wind, and the members have to be designed to meet the greatest stress brought upon them in either case.

In Fig. 90 it is assumed that the wind acts horizontally against the left side of the roof, and that the feet of the rafters are both fixed to the abutments—in which case it is only necessary to reverse the diagram should it be required to ascertain the stresses due to a wind from the right.

The only difficulty met with is that of ascertaining the

amounts and directions of the reactions. It is evident that the pressure $\frac{P}{6}$ which lies between the spaces *a* and

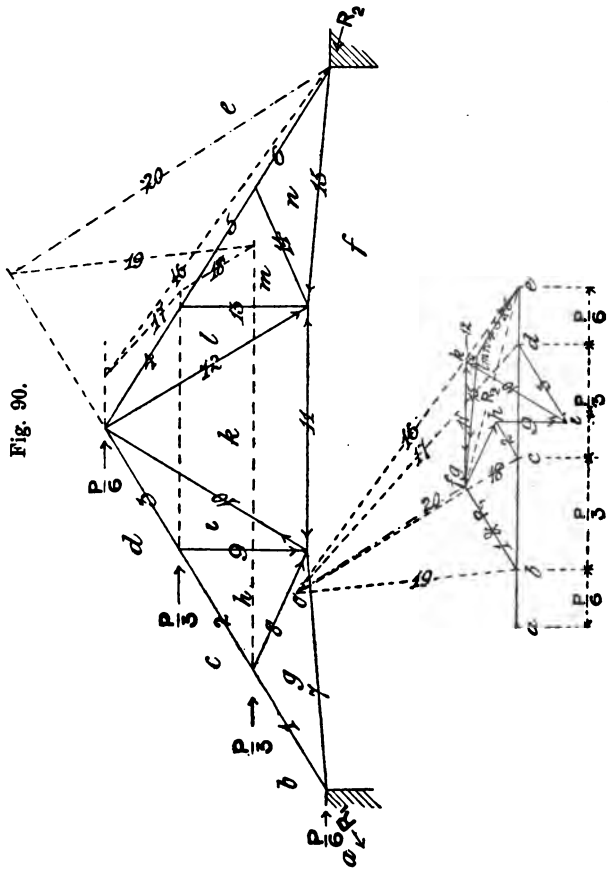


Fig. 90.

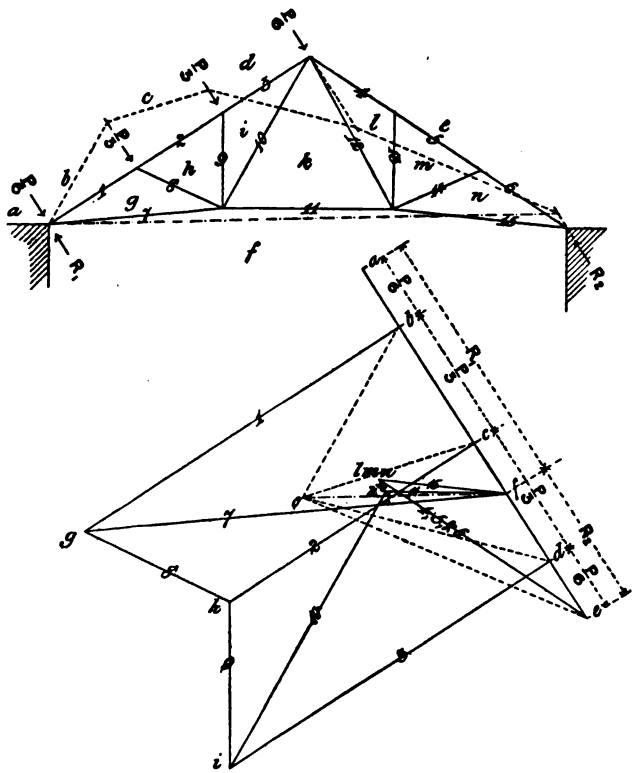
b is resisted directly by the abutment: it may therefore be eliminated from the problem, the other pressures

only being considered. These other pressures clearly place the members 1, 2 and 3 in *tension*, with the result that the abutment must pull against the rafter to the same extent as that to which it is strained.

This gives a reaction *downwards*, in the same direction as the rafter, a sufficient weight having to be provided, attached to the shoe, to supply this reaction. Having thus determined the direction of the reaction of the left-hand abutment, the polar diagram and funicular polygon can be used to ascertain its amount, and also the value, both in direction and amount, of the other reaction. Taking any pole o, join $o\,e$, $o\,d$, $o\,c$ and $o\,b$, numbering these lines 16, 17, 18 and 19 respectively. The funicular polygon must be started from the point where the attachment is made between the roof and the right-hand abutment, for there only is it certain that R_2 will be met, its direction not being known. From that point draw a line 16 parallel to 16 on the diagram, till the line of direction of $\frac{P}{6}$, lying between e and d, be reached. Continue the polygon with 17 and 18 parallel to 17 and 18, till the lines of the pressures lying between d and c and c and b be reached respectively. Then draw 19 parallel to 19 till the direction of R_1 be reached, in the same line as the members 1, 2 and 3. Close the polygon with line 20, and from o on the diagram draw 20 in a parallel direction till a line from b, parallel to the known direction of R_1, be reached in f. Then (the effect of $\frac{P}{6}$ lying between a and b being neglected) $b\,f$ will scale off the value of R_1, and $f\,e$ will denote R_2, both in amount and direction. Once the point f is determined, the remainder is simple enough, upon the usual lines. The directions of the stresses, however, as shown by the arrows round the joints, are instructive. The

members 1, 2, 3 and 10 are in tension, 7, 13 and 14 are unstrained, and all the rest are in compression. This shows that if 11 and 15 be designed as tension rods, they

Fig. 91.



would not suffice to meet the compression brought upon them by a strong horizontal wind. The easiest practical way to meet this difficulty is to add to the weight of the

roof covering in exposed situations, lightly-constructed roofs in such positions being a mistake. The compression in the rafters 4, 5 and 6 is thus increased somewhat, but 11 and 15 can still be retained as ties and used in considerable length.

Except under rare conditions, however, the wind does not act horizontally, but beats either upwards or downwards. When a building is properly closed in, an upward-acting wind, even of considerable power, has but little effect upon the roof, though in open sheds the case is different, and the possibility of an upward eddy has to be taken into account. Downward pressures are common, and exercise their greatest force when acting normally to the slope of the roof. The following is the generally accepted allowance per foot super of surface exposed for different inclinations.

Pitch of Roof.	Normal Pressure of Wind in pounds per foot super.	Pitch of Roof.	Normal Pressure of Wind in pounds per foot super.
5°	5·0	50°	38·1
10°	9·7	60°	40·0
20°	18·1	70°	40·0
30°	26·4	80°	40·0
40°	33·3	90°	40·0

In Fig. 91 is shown a roof exposed to such a normal pressure upon one of its sides. If the heels of both rafters be fixed to the abutments, as is generally the case in small roofs, and as is presumed in Fig. 91, the reactions will both be opposite in direction to the wind pressure. Their values are easiest found by means of the polar and funicular polygons, which are shown in dotted lines, the proportion

of the pressure, $\frac{P}{6}$, which acts directly on the left-hand abutment, being disregarded, as evidently no portion of it is conveyed to the right-hand support. The polygon might in this case have been started from *any* point in the direction of R_1, the reactions and the pressures being parallel, and the same closing line, or one parallel to it, would have been obtained, fixing the same point *f* on the diagram to represent the space *f* on the sketch. The reaction R_2 lying between the spaces *e* and *f* is now shown, both in value and direction, by the line *e f* on the diagram, and R_1 by the line *f a*. The point *f* once ascertained the rest is easy, the stresses upon the various members being found in the usual way.

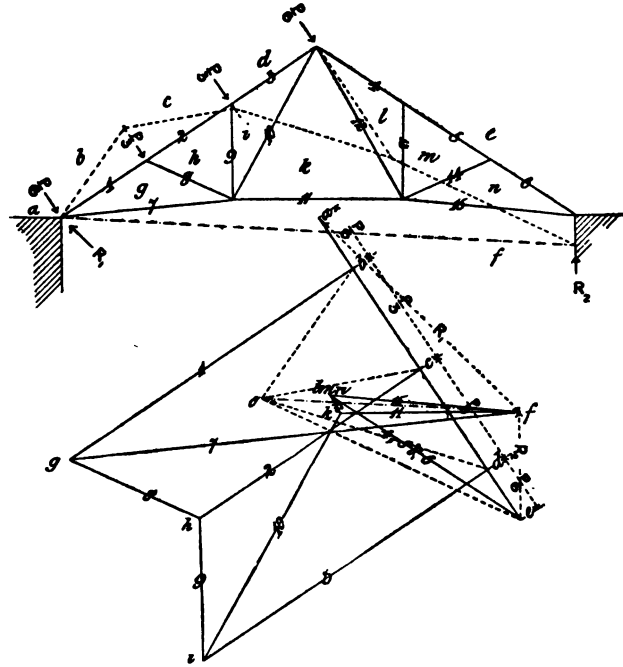
It would have been possible to have ascertained the value of the reactions upon the principle of moments arithmetically; but this is now rarely done save in important structures as a check upon the graphic calculations, which, however, prove themselves by the closing of the diagrams if carefully drawn.

Figs. 92 and 93 are illustrative of cases in which one of the principals only is fixed to its abutment, the other resting upon rollers so as to allow of expansion. In such cases the inclined reaction is all taken up by the fixed end, there being vertical reaction *only* through the rollers. This is a common arrangement in large roofs when the expansion and contraction of the members (if constructed of iron), which is due to variations of temperature, has to be provided against.

In Fig. 92 it is presumed that the principal is fixed to the left-hand abutment, the rollers being on the right. The reaction at the right abutment (R_2), therefore, is vertical; but all that is known of R_1 is that it passes through the point of attachment of the principal and the

left abutment. The funicular polygon may therefore be started from that point with safety, and continued to a vertical line representing the reaction whose direction is known. The closing line, when transferred to the polar diagram, is also drawn to meet a vertical line through *e* at

Fig. 92.

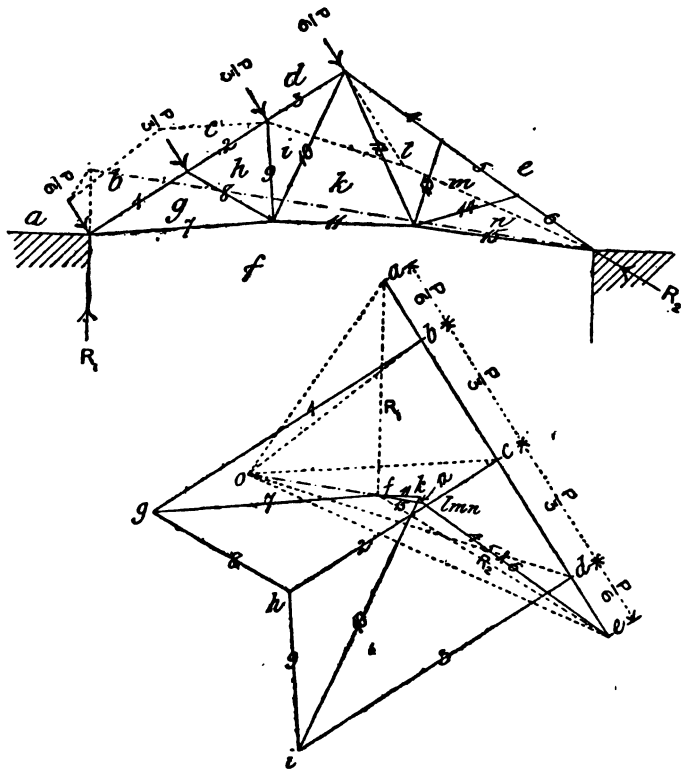


f, giving *e f* as the vertical reaction R_2, the other reaction, R_1, being equal both in direction and amount to *f a*.

The method of ascertaining the reactions in Fig. 93 is precisely similar, save that the funicular polygon is started from the point of attachment of the principal and

the right-hand abutment, and is drawn till a vertical line through the left-hand abutment is reached, giving R_1 in the diagram as a vertical force from f to a, and R_2 as an inclined reaction from e to f.

Fig. 93.



In both cases, once the point f is found on the diagram, and the reactions determined, the rest is plain sailing, and the stresses upon the members, both in direction and

value, can be ascertained as usual, the closing of the diagram proving its correctness.

Fig. 94 shows the combination in one diagram of the stresses produced by a vertical load and a horizontal wind pressure acting concurrently.

The first thing necessary is to determine the actual forces acting at the various points. Between *a* and *b* there acts a horizontal pressure $\frac{P}{6}$ and a vertical load $\frac{W}{12}$. These being combined give a resultant of F_1. From *a* to *b* on the diagram, therefore, set down a line equal to scale, and parallel in direction to F_1. Similarly, F_2 is the resultant of the pressures acting between the spaces *b* and *c*, F_3 the resultant of those acting between *c* and *d*, and F_4 the resultant of those acting between *d* and *e*. From *b* to *c* on the diagram, therefore, a line is drawn equal and parallel to F_2, from *c* to *d* another equal and parallel to F_3, and from *d* to *e* yet another equal and parallel to F_4. The lines from *e* to *f*, *f* to *g*, and *g* to *h* are vertical, being equal and parallel to the loads acting vertically downwards between the spaces *e* and *f*, *f* and *g*, and *g* and *h* respectively.

If the principal be fixed to both abutments, it is evident that F_1, lying between *a* and *b*, and $\frac{W}{12}$ lying between *g* and *h*, are met directly by the resistance of the abutments on which they respectively act; and that they may therefore be neglected in considering the reactions which affect the structure. By joining *g b* on the diagram, the polygon *b c d e f g b* is closed, this representing the external forces acting upon the truss, the line *g b* giving the value of the sum of the reactions both in amount and direction. The direction of the reactions being thus ascertained, a funicular polygon with its reciprocal polar diagram can readily

be set up (as shown by dotted lines) to determine how much of the total reaction $g\,b$ is operative at each abut-

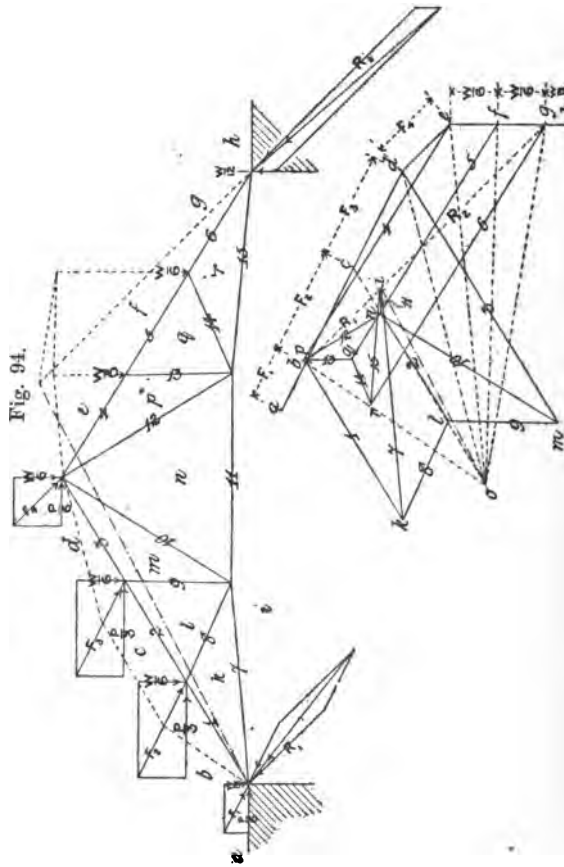


Fig. 94.

ment, $g\,i$ being found to be the value of R_2, and $i\,b$ that of R_1. These reactions are irrespective of those caused by

the forces acting directly upon the abutments, the composition with these being shown on the sketch, the resultants in the parallelograms there given representing the forces which the abutments would have to be designed to meet.

When the point *i* has been determined, the diagram of stresses can be formed in the usual manner, and it will prove its own correctness by closing at the finish. It will be noticed that, in the example taken, sufficient dead load has been assumed to so far counteract the wind pressure as to place the members 11 and 12 in tension. The member 12 would, however, be designed to resist the tension found to exist in 10, in case of the wind acting in the contrary direction to that calculated for; and, the same system being pursued throughout, the roof would be symmetrical in construction, each member being made capable of resisting the greatest stress that would be likely to be brought to bear upon it under any ordinary conditions.

In Figs. 95 and 96 all the necessary work is shown in order to ascertain the stresses upon a segmental roof of the "Warren" pattern, when subject to a depressed wind pressure of 40 lbs. per vertical foot super (or rather to the normal equivalent of 26·4 lbs. per foot super of surface exposed for an inclination of 30°), and to a dead load of 20 lbs. per horizontal foot super. These pressures are compounded at the various joints, as explained above; but the resultants only are shown for the sake of clearness, both direction and value in lbs. being given. For the same reason the lettering and arrows, shown in previous diagrams, is here omitted. The trusses, which are 10 feet apart, are supposed to be fixed at the left-hand abutment, and to rest upon rollers over that at the right, Fig. 95 giving the diagram when the wind is from the left, and Fig. 96 that obtained when the wind is from the right.

The table on page 115 contains the value and denomination of the stresses upon the various members as scaled off from the diagrams, and from this table the members would be designed.

For members which are in tension under all circumstances, such as 12, 15, 18, 21 and 24, light circular rods might be used, containing just enough metal to resist the stress; but in all other members, even those which are commonly called "ties," the possibility of their being placed under compression must be considered. They should either be bars, L or T irons, and must be designed with the aid of the formulæ given in Chapter VII. for columns of various forms. In some of these members both ends are fixed by means of rivets; in others one end is fixed, the other bearing upon a pin; while in others both ends are pinned; and C, 2 C or 3 C has to be taken in the formula used accordingly. The rafters in this example, from their being curved, are not pure struts. In ascertaining the compressions upon them they have been considered as straight from point to point. The stresses thus found upon each, multiplied by the camber in the centre, gives a bending moment, to which the stress moments of the portions of the rafter which are in tension and compression respectively must each be equal, the line taken in the diagram being presumed to lie along the neutral axis.

To complete the design, the shears upon the various pins and rivets have to be considered. It is the modern practice to employ double cover-plate joints, each member being separately pin-connected, in which case the shear upon each pin is equal to the stress in the member it connects, and, being in double shear, each pin presents two sectional areas to resist this shearing stress.

1340 lbs
R2
R1
930 lbs
3900 lbs
3900 lbs
2860 lbs
1340 lbs
R1

TABLE of STRESSES upon the members of the segmental roof shown in Figs. 95 and 96 for a span of 60 feet (rise in centre 17′ 4″ and camber of tie-rod 5′ 5″), when subject to a dead load of 20 lbs. per horizontal foot super and normal wind pressures of 26·4 lbs. per foot super, the trusses being 10 feet apart.

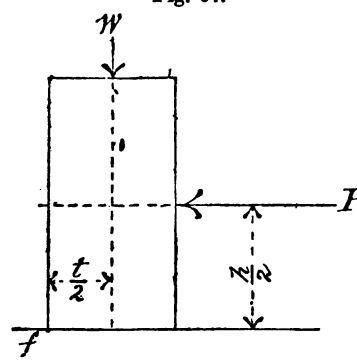
Number indicating Member.	Stress in lbs. (wind from left).	Stress in lbs. (wind from right).
1	+ 13,600	+ 6,475
2	+ 12,550	+ 6,025
3	+ 11,900	+ 6,550
4	+ 11,350	+ 7,275
5	+ 9,525	+ 9,025
6	+ 8,900	+ 9,475
7	+ 8,275	+ 10,050
8	+ 8,725	+ 11,125
9	− 12,900	+ 750
10	− 700	− 1,200
11	− 275	+ 1,200
12	− 13,075	− 475
13	+ 100	− 1,800
14	− 1,675	+ 1,700
15	− 12,275	− 1,925
16	+ 1,350	− 2,400
17	− 3,075	+ 2,400
18	− 10,400	− 3,625
19	+ 2,050	− 2,700
20	− 2,900	+ 2,000
21	− 8,750	− 5,300
22	+ 1,600	− 1,600
23	− 2,425	+ 625
24	− 7,150	− 6,300
25	+ 1,325	− 250
26	− 1,825	+ 225
27	− 5,500	− 6,325

CHAPTER XII.

WALLS AND CHIMNEYS TO RESIST WIND PRESSURE ONLY.

VARIABLE as is the force of the wind, and at the same moment of various intensities at various levels, acting sometimes horizontally and sometimes in eddies, it has been found to be sufficiently accurate for most practical purposes, when dealing with the thrusts on walls and

Fig. 97.



chimneys, to assume it to be a horizontally-acting uniform force. Its value can only be found by experiment in the particular situation where the erection under consideration is to be placed; but as a general rule, and in most situations, a pressure of 40 lbs. per foot super of the surface exposed to its action may be assumed with safety as the

maximum ever likely to occur, though in exposed situations a higher value of 50, 56, or even 60 lbs. per foot super is often calculated for.

This pressure may, too, without sensible error, be assumed to act at the centre of gravity of the surface pressed against, and in a horizontal direction.

In the case of an ordinary rectangular wall of masonry or brickwork, this centre of pressure is found to lie at a point half-way up the height of the wall.

Suppose, for example, that the wall shown in Fig. 97 is hinged at the point *f*, and so capable of overturning at the joint at its base, the resistance of the mortar not being taken into account, then the moment of pressure tending to overturn the wall is $P \times \frac{h}{2}$, the value of P being the pressure of the wind per foot super × the surface exposed in feet.

The moment of stability, tending to cause rotation in the opposite direction, and so to preserve equilibrium, is equally obviously $W \times \frac{t}{2}$, the value of W being the weight of the wall.

Therefore, when there is just equilibrium and no more—

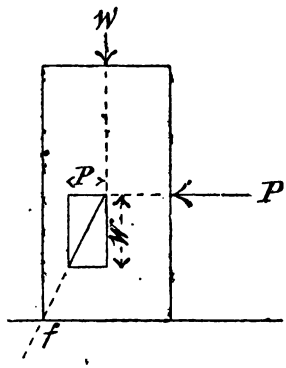
$$W \times \frac{t}{2} = P \times \frac{h}{2}.$$

Of course, under normal circumstances the moment of the weight will exceed the moment of pressure (P being taken at maximum), and then any excess of tendency to rotate caused by W is met and equated by the reaction of the earth.

It is obvious also, that when the above equation exactly holds, if the two forces W and P be set out, as shown in

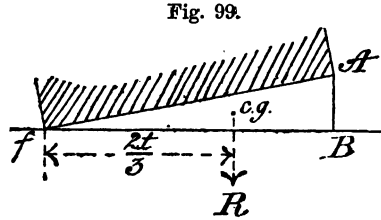
Fig. 98, to scale in the directions in which they act, from the point where their directions cross, and a parellelogram of forces be found, their resultant will, if continued, exactly pass through the point *f*.

Fig. 98.



Another consideration now forces itself upon us. As soon as the moment of pressure exceeds the moment of weight, and rotation commences around *f*. the resistance to tension of the mortar in the joint comes into play. This it does, as shown in Fig. 99, being most strained where the joint opens most, at A B, while it is *nil* at *f*. At A B the tension must not exceed what the mortar is calculated to bear safely, so that the whole surface upon which the wall rests may be considered as having a layer of mortar having *half* this power of resistance spread over its surface, and this resisting force to tension may be considered to be collected at its

Fig. 99.



centre of pressure, which is the centre of gravity of the triangle A *f* B, and to act vertically, helping W to counteract the overturning power of P.

The moment of resistance of the mortar (R), thus becomes $R \times \frac{2t}{3}$ (see Fig. 99), R passing vertically through the c.g. of the triangle A *f* B, and the total equation when there is just equilibrium is now—

$$\left(W \times \frac{t}{2}\right) + \left(R \times \frac{2t}{3}\right) = P \times \frac{h}{2}.$$

When considering a wall of any length it is usual to calculate for one foot run of it only, and then either W, R, P, t or h can be found if all the others are known (and, as a rule, in practice it is t which is required, h being fixed by circumstances), for W is dependent upon t and h, R upon t, and P upon h. W, for example, is equal to the weight per foot cube of the material to be used $\times t \times h \times$ 1 ft. run of wall (t and h being taken in feet), and this can be substituted for W in the equation if necessary. Similarly P is equal to the wind pressure per square foot $\times h \times$ 1 ft. of wall (h being taken in feet), and R = the safe resistance *per square inch* of the mortar to be used $\times t \times$ 12 in. run of wall (t being in this case taken *in inches*).

It is, however, usually considered that the mortar should never be allowed to come into tension, and to avoid this, and produce an uniformly varying intensity of pressure upon the joint, the resultant of W and P must pass through a point *g*, situated at a distance of $\frac{t}{3}$ from *f*, or of $\frac{t}{6}$ from the middle of the joint, in long walls or solid blocks of masonry which are rectangular or square on plan (see Fig. 100). The moments then have to be taken about this point *g*, and the equation of equilibrium becomes—

$$W \times \frac{t}{6} = P \times \frac{h}{2}.$$

For blocks of masonry of other plans, the following are the positions at which the point g is taken—

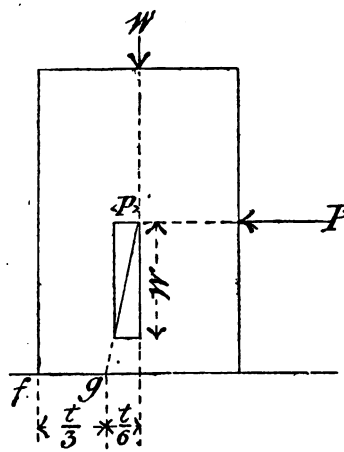
Solid circles: $\frac{3\,t}{8}$ from f, $\frac{t}{8}$ from middle of joint.

Thin hollow squares: $\frac{t}{6}$ from f, $\frac{t}{3}$ from middle of joint.

Thin hollow circles: $\frac{t}{4}$ from f, $\frac{t}{4}$ from middle of joint.

These two last values apply to square and round factory chimneys.

Fig. 100.



Of course, when dealing with such structures as tall chimneys, in which the ring of masonry diminishes in thickness from the bottom upwards, the base course of each thickness has to be taken as a point of calculation, and as many separate calculations made as there are differences in thickness.

In a structure which is not rectangular on plan, too, the

wind is found *not* to act as if its full force were exerted against a vertical plane through the axis of the structure, and at right angles to its direction. It acts thus against surfaces which directly oppose it; but the canted sides (which are generally similar on either side of the axis) offer less resistance to its action, and it may in practice be considered that the wind possesses only half its normal intensity against such sides, supposing them, as just said, to be symmetrically placed.

Carrying this to its logical conclusion, in the case of a structure which is circular on plan, the pressure of the wind against it is only half what it would be against a vertical plane drawn through the axis of the structure.

CHAPTER XIII.

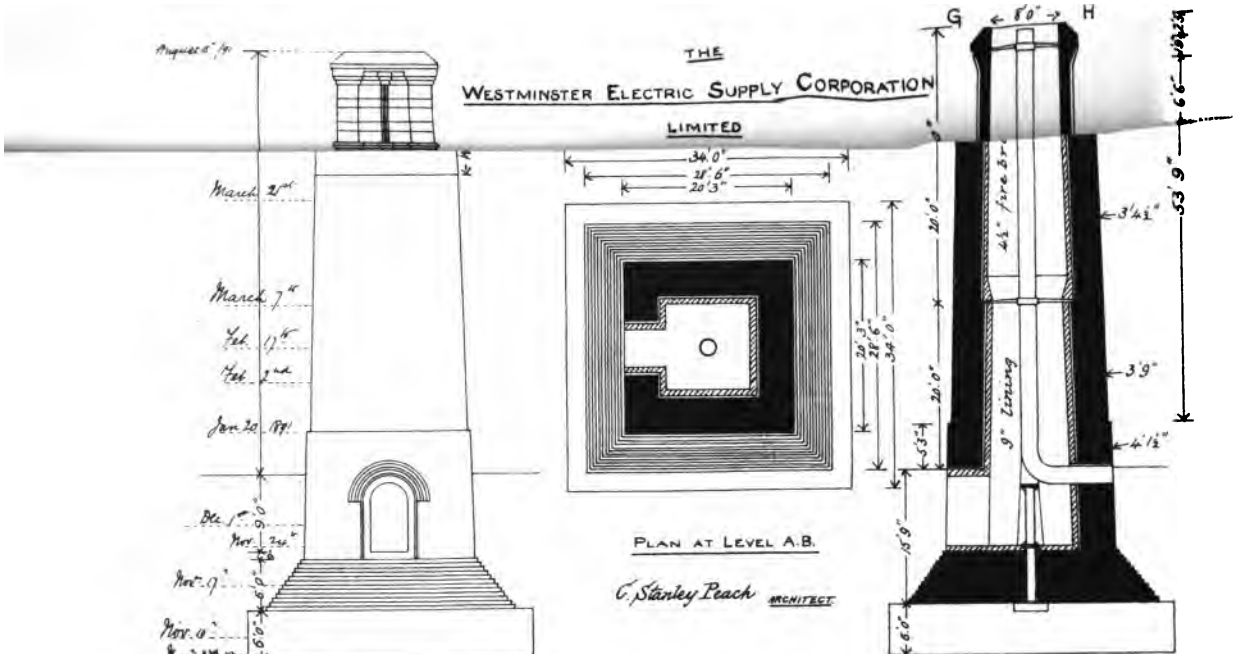
THE CONSTRUCTION OF A FACTORY CHIMNEY.

THROUGH the kindness of Mr. C. S. Peach, in lending a drawing for illustration and in freely supplying information, it is possible to introduce a chapter here dealing with the practical construction of a large chimney.

That selected for illustration, and shown in the Plate, is a square shaft, 160 ft. high, and of 8 ft. side internally at top, having a double batter of 1 in 18·35, and built according to the County Council regulations, which require that the thickness of walls shall increase 4½ ins. at every 20 ft. from the top downwards. This, of course, obviates to a very large extent indeed the necessity for calculating the resistance to wind pressure, but it is very doubtful whether the result is so satisfactory as that which would be secured were theoretical rules properly followed. However, there are the regulations, and at present there is nothing for it but to carry them out.

Strong bricks of the London stock class have been found to be best for the walls, set in lime mortar, and *not* in cement—the latter being too rigid to allow of swaying under a high wind. Tall chimneys and tall steeples always rock, and it has been found by experience to be a mistake to endeavour to stop their doing so.

The lining should be of the very best firebrick procurable, laid in fireclay; but as little of the latter should be



J. Akerman, Photo-lith. London

used as possible—only just sufficient, in fact, to cause the bricks to adhere.

Between this lining and the walls there should be an air space of about 2 in., air being a non-conductor of heat, so that however greatly the lining may become heated, the walls shall be subjected to no excessive temperature. The lining is thus, also, left free to expand and contract without injuring the structure, to which it is *not* bonded, the nearest approach to a bond being formed by introducing headers into the lining, which project into the cavity until they almost touch the outer walls, though they are not allowed quite to do so.

Many designers introduce rows of hoop-iron bond into their walls, but others do not, being afraid of their expanding under high temperature. As already pointed out, however, the walls themselves should never be heated to any dangerous extent, and the objection to the use of bond accordingly disappears.

The lining, it will be noticed, is stopped, and should be stopped, at a little below half the height of the shaft. It is only needed where the temperature is highest; and, moreover, an expansion-chamber is wanted at the half-height, or thereabouts, or else there will not be complete cremation of the fumes. By omitting the lining, therefore, at that spot, the internal dimensions of the shaft are increased, and a sufficient expansion-chamber is provided.

The external form of the chimney, too, is not entirely without its practical effect. By the architect it will be noticed that Mr. Peach has succeeded in giving some architectural character, and this without great additional cost, to even so prosaic an erection as a factory chimney; but a projecting cornice could not be indulged in, from its tendency to interrupt, if not destroy, proper draught.

The steam-exhaust, too, is an important consideration: it has to be carried upon a solid stanchion and stayed at intervals; but the stays must *rest* only upon the off-sets of the brickwork, and on no account be built in, as, being of metal, the exhaust-pipe tends to expand quite differently from the brickwork, and if there were rigid connection there would be risk of something going wrong.

As far as the plan of the shaft is concerned, the square form has been found in practice to be the least satisfactory, giving the least draught. The best form seems to be an irregular octagon, or a square with the angles just taken off, possessing almost the capacity of the square, but without its dead angles. A regular octagon is nearly, but not quite, so good.

It is a most important point to attend to, that the building shall be slowly carried out. Two feet per diem should be the utmost allowed, while it is better if the foundations can be allowed to stand for some months, or even a year, before the superstructure is added. In the example given in the Plate it will be seen that the rate of progression was very slow indeed, the heights reached at different dates having been noted as the work proceeded.

CHAPTER XIV.

RETAINING WALLS SUBJECT TO EARTH PRESSURE.

In dealing with retaining walls, the main difficulty is to ascertain the pressure which is bearing against them, tending to cause them to overturn; and there are two main cases to be considered, those of earth and those of water pressures. By earth pressure is meant the thrust due to normally moist earth, as if it be wet or water-logged through lack of proper drainage at back of, and weep-holes through the wall, the thrust approximates to that from water. This warning is the more necessary, as water pressures, as will be presently shown, are considerably greater than earth pressures, and therefore a wall designed to be amply safe when retaining earth may well overturn when that earth becomes charged with water, for which no means of escape has been provided.

Different classes of earths have very different effects upon a retaining wall, this depending upon the *natural slope*, or *angle of repose*, which will be assumed by a heap of this earth when resting upon the ground. The following table, taken from the 'R.E. Aide-Mémoire,' gives the angle of repose of most earths met with in practical work, but where it should happen not to apply, the value can be readily found by a simple experiment.

It will thus be seen that all particles of earth lying within the angle of repose are naturally held in equi-

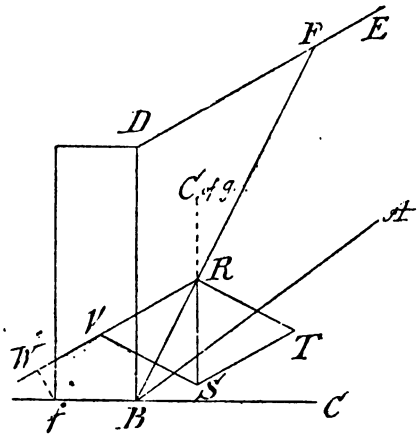
Class of Earth.	Angle of Repose.	Ratio of Base of Slope to Height.
Fine dry sand	37 to 31°	1·3 to 1, to 1·6 to 1
Sand, wet	26°	2 to 1
„ very wet	32°	1·6 to 1
Vegetable earth, dry	29°	1·8 to 1
„ „ moist	45 to 49°	1 to 1, to ·87 to 1
„ „ very wet	17°	3·27 to 1
„ „ punned	66 to 74°	·45 to 1, to ·28 to 1
Clay, dry	29°	1·8 to 1
„ damp, well drained	45°	1 to 1
„ wet	16°	3·5 to 1
Gravel, clean	48°	·9 to 1
„ with sand	26°	2 to 1
Loose shingle	39°	1·2 to 1
Peat	14 to 45°	4 to 1, to 1 to 1

librium, and are in repose, needing no retaining wall to support them. Particles lying directly beyond this slope, moreover, tend to roll down it, but with a motion and force greatly impeded by friction; while, assuming the back of retaining wall to be vertical, the particles next to the wall tend to fall vertically downwards. Between these two extremes there are all degrees. In any event, however, it may be conceded that the pressure against the back of the retaining wall is that due to the prism of earth above the angle of repose, and that this is resisted partly by the friction of the earth below, and partly by the retaining wall.

Of course it is possible for the surface of the retained earth to be either horizontal or inclined at any angle within the angle of reposed—or even to be depressed,

though this last case is rare and scarcely ever calculated for. In the case of a horizontal or surcharge surface, however, there are widely different opinions held by various authorities as to the direction in which the resultant pressure of the active prism of earth will be exerted against the wall. Poncelet holds that it is always horizontal, Rankine that it is always parallel to the actual

Fig. 102.



surface of the earth, and Scheffler that it is always parallel to the natural slope. Poncelet's direction gives the greatest moment of overturning, Rankine's is mathematically true for perfectly dry earth, while Scheffler's has been found in practice, though giving the least moment of overturning, to be reliable. Rankine's, however, is the theory most generally adhered to—in England, at any rate.

Returning now to the consideration of the value of the thrust, and taking the case shown in Fig. 102 as a gener-

ally typical one, the wall being rectangular, the earth surcharged to the plane D E, which is within the natural slope, the angle of repose being A B C, it is seen that it is the prism of earth E D B A which is active to cause thrust against the wall. The earth within this prism has, as has been already shown, exceedingly various tendencies to press outwards against the back of the wall D B, and it is a usual and sufficiently correct custom to bisect the angle D B A by the line B F, and to consider the triangular prism of earth D B F as being homogeneous, and to be acting as a wedge against the two plane surfaces D B and B F, which are devoid of friction, offering only dead resistance.

The weight of, say, 1 ft. run of this prism D B F of the earth under consideration is now ascertained from the following table:—

WEIGHT PER CUBIC YARD OF DIFFERENT SOILS.

	cwt.		cwt.
Dry peat	7½	Common gravel	27
Wet peat	15	Wet sand	28
Top soil	20	Gravelly clay	30
Dry sand	22	Rough water gravel	34
Common earth	24	Grey chalk	36
Sandy loam	24	Sandstone	37
Marl	26	Shale	39
Clay	27	Limestone	40

This weight is collected at the centre of gravity of the triangle D B F, and acts vertically downwards until it impinges at R upon the line B F. At this point it is set vertically downwards, as R S, to a scale of weights, and is resolved out according to the parallelogram of forces into its two component forces, R V acting parallel to the surface of the ground, and R T acting perpendicularly to the line B F. R V is the measure to scale of the force

tending to overturn the wall, acting in a direction parallel to the surface of the earth, and impinging upon the back of the wall at a distance from its base equal to one-third of its height, as is shown by the construction. The moment tending to overturn the wall about the point *f* is therefore R V × W *f*, the distance W *f* being measured by dropping a line from *f* perpendicularly to the line R V continued.

This tendency to rotate of course has to be met by the tendency to rotate in the opposite direction, as measured by the moment of the weight of the wall, also taken about *f*.

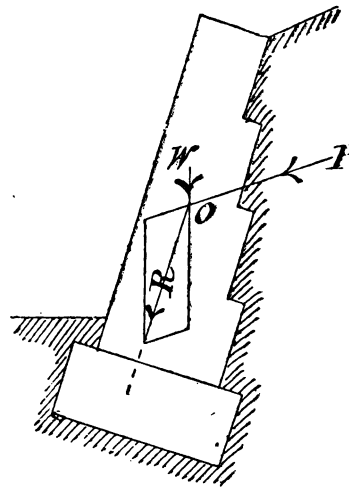
Had Poncelet's theory been adopted, the line R V would have been drawn horizontally; while according to Scheffler's, it would have been laid down parallel to B A, the natural slope. This would not only have altered the length of R V, but also that of the line W *f* drawn perpendicularly to it from *f*, and so have materially modified the result.

So far as the weight of the wall is concerned, this is usually easily ascertained; but it should be borne in mind that if the back be either battered or built in off-sets, all earth lying vertically above the base of the wall acts to assist the wall. Its weight should be added to that of the wall, and concentrated at the c.g. of wall and of this earth combined. At the same time, the back of the wall against which there is earth pressure (R V, as found above) must be considered to be a vertical line drawn upwards vertically from the back of the base of the wall.

As to the shape which the section of the wall should assume, this is largely a matter of convenience. The rectangular vertical form is possibly the most common in most work; but its value can be greatly increased by

sloping it backwards, so that a vertical line through its c.g. falls further within the base. External buttresses are often used, and, if not too far apart, are valuable additions as widening the base, overturning having then to take place round their outer edge instead of round that of the wall. Where circumstances do not permit of their introduction, internal counterforts are often used instead; but

Fig. 103.



they can hardly be considered to be so valuable, acting in tension instead of compression. Battered faces, internal or external, are economical in masonry or concrete, but difficult to bond well in brickwork, if the other face be kept vertical. A stepped back with vertical face has much to recommend it; but possibly the best form for brickwork is that shown in Fig. 103. In this the pressure P, as already found, and the weight of wall W meet at O,

where a parallelogram of forces is formed, the resultant R of which passes within the centre third of base. The concrete is set at right angles to the direction of R, and the face of wall at right angles to concrete and parallel to R, the back being stepped, but kept parallel to the face.

CHAPTER XV.

RETAINING WALLS SUBJECT TO WATER PRESSURE.

WATER, being a fluid, has an angle of repose of zero, and cannot be surcharged. This is the essential difference between it and earth, while its weight varies so little according to its temperature and ingredients as to be for all practical purposes always 62·5 lbs. per cubic foot. Thus to calculate water pressures is easy to anyone who has mastered earth pressures, the same rules being applicable.

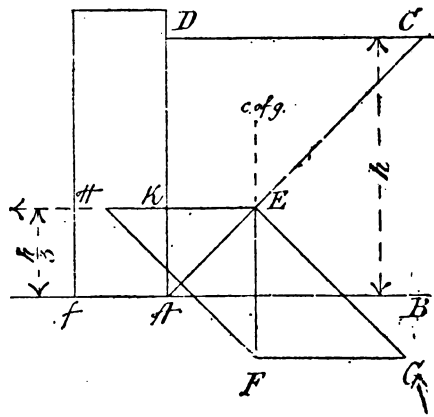
Reference to Fig. 104 will show that A B being the bottom, and D C the top of the water in a reservoir, retained by a vertically-backed wall at D A, and the natural slope being horizontally along the line A B, the complementary angle is the whole angle B A D, and this being bisected by the line A C, according to the rules laid down in the last chapter, the triangular prism of water which is active to cause thrust is that shown in section by the triangle C A D. The weight of this is collected at its centre of gravity, whence it acts vertically downwards, impinging upon the line A C at E, where it is resolved into two forces, E H and E G, of which E H is the force tending to overturn the wall, to the scale to which the weight A F of the triangular prism C A D was set downwards from E.

A little examination of the diagram will show that E F

being vertical, H E horizontal, and E G at right angles to A G, which itself bisects the right angle D A B, therefore the angles E H F and E F H are both angles of 45°, and, therefore, that H E = E F. In other words, the force E H tending to overturn the wall is equal to the weight of the triangular prism of water, C A D.

It will further be seen that the distance A K at which this force acts from the base of wall is exactly one-third of the depth of the water A D, for the vertical line through

Fig. 104.



the c.g. of the triangle C A D being parallel to A D, and impinging upon A C at E, makes $A E = \frac{A C}{3}$, and similarly, E K being parallel to C D, therefore A K : A D :: A E : A C, which proves the proposition.

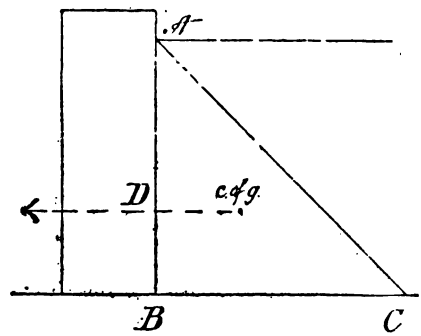
It is more convenient, and quite as correct in practice, to reverse the prism, as shown in Fig. 105, and to say didactically that the pressure against the back of the wall A B is equal to the weight of the 45° triangular

prism of water A B C, acting horizontally through its centre of gravity, and thus impinging upon A B at D, D B being equal to $\frac{A B}{3}$.

In one very marked respect, however, does water act in a different manner from earth. Being a fluid, it always presses perpendicularly against the surfaces opposed to it.

This is a most valuable property, and is largely taken advantage of in designing retaining walls by battering

Fig. 105.



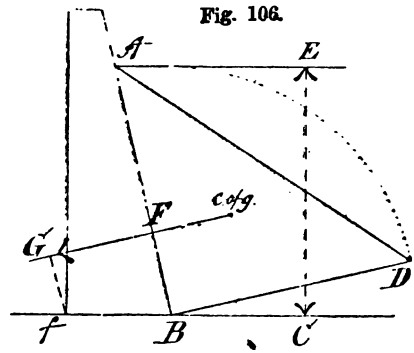
them internally. The effect of thus battering a simple wall is shown in Fig. 106. The triangular prism is formed by drawing B D at right angles to the battered face A B, and equal in length to the vertical depth C E. The c.g. of the triangle A B D thus formed is found, and the weight of the triangular prism acts through it, along the line F G, perpendicularly to the inclined side A B, forming the pressure which tends to overturn the wall, and acting at the distance *f* G round the point *f* of overturning.

It will thus be seen that, although the pressure of

water against the side of the wall is slightly increased, A B being greater than the vertical depth C E, the moment of overturning is largely diminished, f G being much less than $\frac{C E}{3}$.

It is not altogether unusual to build a wall as shown in Fig. 107, with the upper portion vertical and the lower battered. In such an instance two calculations are necessary—for the upper portion only, as rotating about f; and for the whole wall, upper and lower portions com-

Fig. 106.

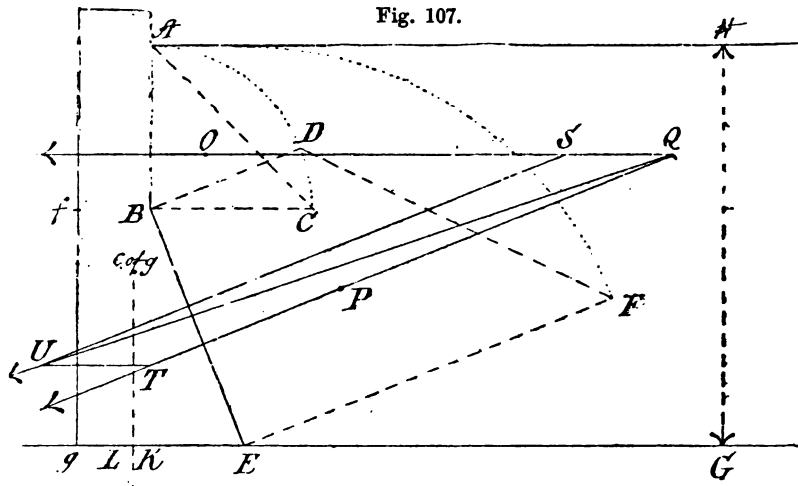


bined, as rotating about g. The upper portion A B may be considered to be pressed against by the weight of the triangular prism of water A B C acting horizontally through its centre of gravity, O. The lower sloped portion B E has a curiously shaped prism of water acting against it, represented by the polygon B E F D, of which B D is drawn at right angles to B E, and is equal to the depth B A of the point B from the surface of the water; while the line E F is also drawn at right angles to B E, and is equal to the depth H G of the point E from the surface of the water. The weight of this prism, B E F D,

is collected at its centre of gravity, P, and acts perpendicularly to the inclined surface B E.

To obtain the resultant pressure against the whole wall, it is necessary to combine the pressure of the triangular prism A B C passing horizontally through O, and the pressure of the irregular prism B E F D passing through P, by continuing their directions back till they meet in Q, and there setting up a parallelogram of forces

Fig. 107.



Q S U T, of which Q S represents the action of the triangular prism in magnitude and direction, and Q T similarly represents the action of the irregular prism, and of which Q U is the resultant.

This, of course, has to be combined with the weight of wall acting through its centre of gravity vertically downwards to obtain the eventual resultant, which must pass within the base of the wall as against actual overturning. As already said, however, it should pass well within the

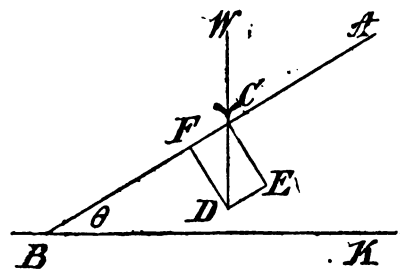
base of the wall to secure anything like uniform variance of pressure, but how far within is not always easy to determine. In practice, however, it may be said that if the vertical line through the centre of gravity of the wall cut the base at K, the final resultant should pass between K and L, the line K L being one-third of the line K *g*.

CHAPTER XVI.

RESISTANCE TO SLIDING.

FAILURE by sliding, though not very common, is yet possible and of occasional occurrence, and so must be provided against. Of course, were all substances perfectly smooth, and so perfectly frictionless, sliding would inevitably take place, and the more nearly two adjacent

Fig. 108.



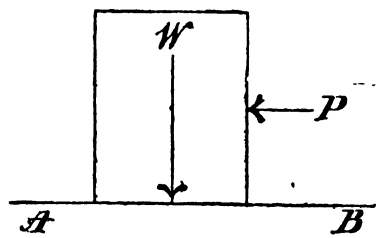
surfaces approximate to perfect smoothness, the greater is the possibility of their sliding upon one another.

Suppose W to be a mass of one material resting upon a wedge A B K of another, the wedge being such that sliding is just upon the point of taking place, the angle A B K or θ being the angle of repose between the two substances. The weight of the mass W can be represented to scale by the vertical line C D, and under these

circumstances it is being held in position by the normal resistance of the wedge A B K, as represented by the line C E at right angles to A B, and by the friction, as represented by the line C F, along A B—the lengths of these lines being determined by forming a parallelogram of forces round C D, resolving it into forces C E and C F, normal and parallel respectively to A B.

F C is the value of the friction between the two substances under consideration, which has to be overcome before sliding can take place. Under the present circumstances, it is just met and equated, by the component of W

Fig. 109.



which acts along A B, and its value is constant for these two substances, being equal to the normal component C E, multiplied by the tangent of the angle of repose (tan. θ).

This is readily proved, for the angle D C E is equal to the angle θ, and D E = F C.

Therefore
$$\frac{D E}{C E} = \text{tan. } D C E$$
$$= \text{tan. } \theta.$$
$$\therefore D E = C E \text{ tan. } \theta.$$
$$\therefore F C = C E \text{ tan. } \theta.$$

Tan. θ is known as the *coefficient of friction*, and is a

constant multiplier for any two materials, the angle of repose between which is known.

Thus, if W be the weight of a block of one material resting upon a horizontal surface, A B, of another, the coefficient of friction between the two materials being known, and the normal component of the weight being in this case equal to the weight itself, then the friction is represented by—

$$W \times \text{coefficient of friction},$$

and the pressure P, parallel with the surface A B, which is necessary to overcome this friction and cause sliding, must consequently be greater than W × coefficient of friction.

The following table gives the generally accepted angles of repose and their tangents (or the coefficients of friction):—

	Angle of Repose.	Coefficient of Friction.
Masonry and brickwork, dry	31 to 35°	·6 to ·7
Ditto with wet mortar	25½°	·47
Ditto with slightly damp mortar	36½°	·74
Ditto on dry clay	27°	·51
Ditto on moist clay	18¼°	·33
Wood on stone	22°	·4
Metals on stone	35 to 16⅔°	·7 to ·3
Wood on wood, dry	14 to 26½°	·25 to ·5
Metals on metals, dry	8½ to 11½°	·15 to ·2
Smoothest and best greased surfaces ..	1¾ to 2°	·03 to ·036

CHAPTER XVII.

ARCHES.

THE theory underlying the correct designing of arches can be expressed in very few words—the line of resultant pressures should in all cases pass within the centre third of the arch ring. In practice, however, it is found that in ordinary cases it is sufficient if this line of resultant pressures pass within the middle *half*, instead of the middle *third*, of the ring; and, of course, in addition to this, the material of which the arch is built must be sufficiently strong to withstand these resultant pressures safely. The problem is, therefore, to determine the value and direction of these resultant pressures.

In the case of a symmetrically loaded arch it is clear that the thrust at the crown is a horizontal thrust. Its true value, however, is difficult to determine, varying according to where within the middle third of the arch ring it acts. In Fig. 110 it is shown acting at the outer, and in Fig. 111 at the inner extremity of the ring; its true position is, in all probability, somewhere between these two extremes. Half of the arch only is shown in either diagram, for the sake of clearness, and in each the weight resting upon the half arch is concentrated at its centre of gravity, acting downwards vertically as lettered, W.

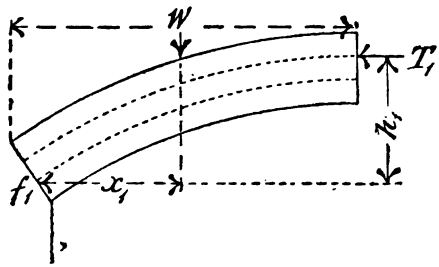
Taking the extreme case shown in Fig. 110, the value of the thrust T_1 is determined by taking moments about

the point f_1 at the abutment extremity of the dotted outline of the *inner* side of the middle third of the arch ring.

Then $$T_1 \times h_1 = W \times x_1 ;$$

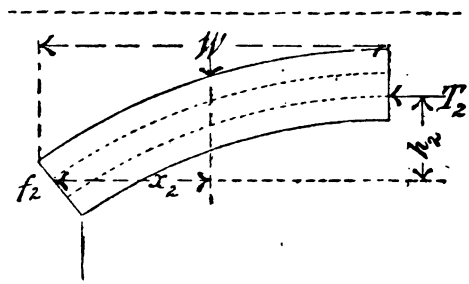
and therefore $$T_1 = \frac{W \times x_1}{h_1}.$$

Fig. 110.



Taking the other extreme case shown in Fig. 111, the value of the thrust T_2 is determined by taking moments

Fig. 111.



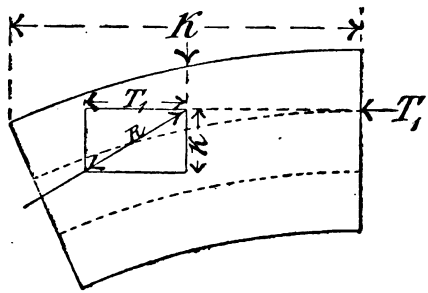
about the point f_2 at the abutment extremity of the dotted outline of the *outer* side of the middle third of the arch ring.

Then $$T_2 \times h_2 = W \times x_2;$$

and therefore $$T_2 = \frac{W \times x_2}{h_2}.$$

Taking either of these extremes, it is treated as shown in Fig. 112. The thrust T_1 is compounded with the weight K of the voussoir nearest the centre of the arch and all it carries, acting through their combined centre of gravity. The resultant R is found, and is produced to cut the joint between this voussoir and the next, the point where it does so being one point in the line of resultant

Fig. 112.



pressures; and consequently it should lie within the centre third of the arch ring.

This resultant is similarly compounded with the weight of the next voussoir and what it carries, a fresh resultant being found and produced to cut the next joint and so on. The points where the resultants cut the joints to which they relate are all points in the line of resultant pressures, and should all fall within the middle third of the arch ring. If they do so, the arch is stable under one of the extreme cases, and no further inquiry is needed; if not, the other extreme case must be tried. If this also result unsatis-

factorily, it will either be evident at once that the voussoirs must be deepened, or that there is some intermediate case which will give a satisfactory result; and in either event the same process of further trial must be continued until a resultant line be found which lies wholly within the middle third of the arch ring.

It will be noticed that a vertical joint is assumed at the centre of the arch, and this is done for convenience of calculation. As a matter of fact, the voussoirs assumed for calculation need not be those used in actual practice, a brick arch of several rings of brickwork, for instance, being treated for calculation as of one ring only of the total depth, and as cut into some half a dozen or dozen voussoirs only. It is well, however, where possible, that the joints between the voussoirs should be at right angles, or nearly so, to the resultant pressure which impinges upon each.

In the case of an unsymmetrical load—by no means an uncommon occurrence in buildings—the horizontal thrust is found to be under the centre of gravity of the loads. It is there where the thrusts T_1 and T_2 for the two main trial cases have to be ascertained by taking moments round f_1 or f_2, as already explained. Beyond this the calculations are identical.

CHAPTER XVIII.

BUTTRESSES.

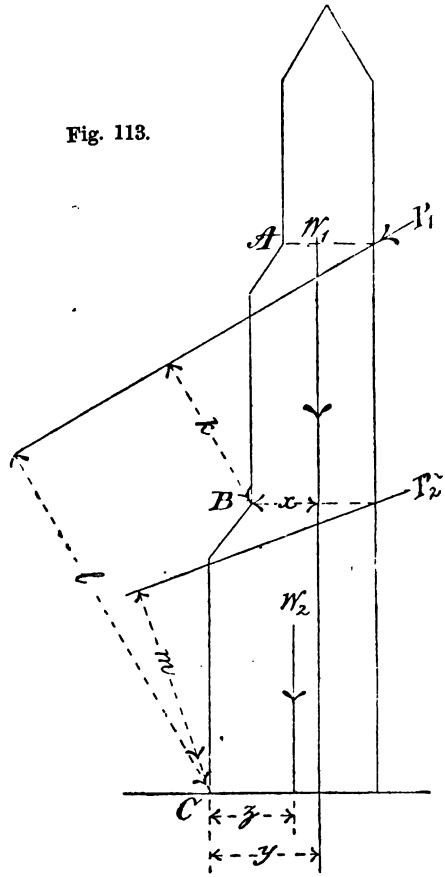
In considering the question of buttresses, it is only necessary to depend upon principles already fully enunciated. The thrusts to which they are subjected once determined, the overturning power which these possess is readily measured by taking moments, just as is done in the case of walls, about their outer edges—and, to obviate overturning, this power or tendency must be at least equated or exceeded by the power of the buttress itself, measured by the moment of its weight about the same outer edge, to resist this overturning. Thus, no special difficulties are presented.

To take one special case, the ordinary one shown in Fig. 113 should be quite sufficient to explain all that is necessary. P_1 is supposed to be the ascertained thrust from an arched flying buttress, and P_2 to be that from the principal of an aisle roof. Of course, at the point A there is no overturning tendency. The weight of the pinnacle above this point, which pinnacle may be as ornate as may be desired so long as it be heavy enough to do its work, is only needed as part of the weight of the buttress above B, acting as W_1 collected at its centre of gravity, with the leverage x, to counterbalance the thrust P_1 acting with the leverage K. Thus, taking moments about B_1—

$$W_1 \times x, \text{ must be greater than } P_1 \times K.$$

Similarly, supposing W_2 to be the weight of the portion of the buttresses lying between B and C, and to be

Fig. 113.



located at its centre of gravity, then the moment of overturning about the point C becomes—

$$P_1 \times l + P_2 \times m,$$

and the moment of stability about the same point is—

$$W_1 \times y + W_2 \times z.$$

Again, the moment of stability must at least equal, and should exceed, the moment of overturning, and, as explained in an earlier chapter, to secure safety the points B and C, around which moments are to be taken, should be fixed at one-third of the thickness of the buttress from its outer edge.

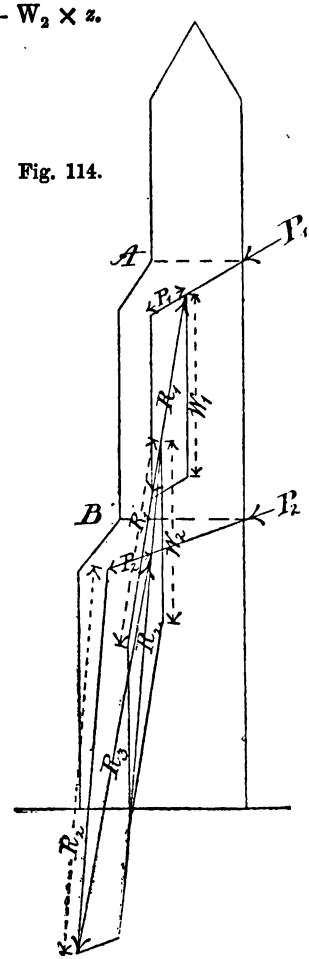


Fig. 114.

It will readily be seen that, by trial, the problem can equally well be solved diagrammatically, as shown in Fig. 114.

P_1 and W_1 are combined where they meet, and their resultant R_1 found. Then this is combined with W_2, and a fresh resultant R_2 found, which is again combined with P_2, and a third resultant R_3

found. Then R_1 must pass within the joint at B, as against overturning, or within its middle third as for safety, and similarly R_3 must pass within the joint at C, as against overturning, and within its middle third for safety.

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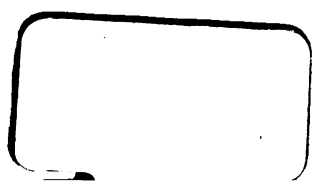

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